

Contribution to State Estimation of Semilinear Parabolic Distributed Parameter Systems with Applications to Transport Reaction Systems

Transport–reaction systems are described by semilinear parabolic partial differential equations (PDEs) and are fundamental in applications where diffusion processes must be considered explicitly. The state estimation problem on the basis of some in-domain distributed measurements is non-trivial. In this work we address this problem for a certain class of transport-reaction systems. To achieve this task, we propose observer design strategies in the frame of both early and late lumping approaches.

Regarding the early lumping approach for the state observer design, we use the Method of Weighted Residuals (MWR), that encompasses the orthogonal collocation method, to derive an approximate reduced-order model, expressed as a set of ordinary differential equations (ODEs) subject to algebraic constraints. Then, a Lyapunov-based design method is proposed for the reduced-order model which provides sufficient design conditions in terms of standard linear matrix inequalities (LMIs) aiming the exponential convergence of the estimation error with a prescribed decay rate. The observer performance is further improved through an offline optimal sensor placement algorithm considering a parameterized reduced-order output matrix.

Concerning the late lumping approach, firstly, we studied the operator semi-group representation which lead us to the use of the spectrum-decomposition properties related to parabolic differential operators. Thus we aimed at obtaining sufficient state observer synthesis conditions based on the local Lipschitz properties of the reaction rate vector functions considering a modal output injection gain. Secondly, a Lyapunov based design method is proposed for the stabilization of the estimation error dynamics. The approach uses positive definite matrices to parameterize a class of Lyapunov functionals that are positive in the Lebesgue space of integrable square functions. Thus, the stability conditions can be expressed as a set of LMI constraints which can be solved numerically using sum of squares (SOS) and standard semi-definite programming (SDP) tools.

Throughout the chapters of this thesis, all of these proposed techniques and methods are applied and tested numerically to the representative cases of biochemical tubular reactor processes. Simulation results support the effectiveness of the suggested designs.

Finally, the COVID-19 spread monitoring problem is addressed in the application part of this thesis. In particular, we tackle the state estimation of the compartmental model based on partial differential equations (PDEs) which describes the spread of the infectious disease in a host population. A Lyapunov based design method with SOS and polynomial parameterization of the decision variables is used to derive a SDP problem whose solution provides the injection gains of the Luenberger type state observer, Numerical experiments are presented to illustrate the method efficiency.