Abstract

Matrix factorizations (MF) are standard techniques in linear algebra that approximate a data matrix as the product of two smaller matrices whose inner dimension is called the rank of the factorization. A general assumption in MFs is the nonnegativity of the input matrix which led to the development of nonnegative matrix factorization (NMF) models, where the factors are constrained to be nonnegative as well, motivated by several real-world applications. NMF allows one to express the columns of the input matrix, which generally represent data points, as linear combinations of a small number of latent features.

In the first part of this thesis, we introduce near-convex archetypal analysis (NCAA), a flexible extension of archetypal analysis, a well-known NMF variant, which has an interesting geometric interpretation and performs competitively with minimum-volume (minVol) NMF.

In the second part of this thesis, we study deep MF, that is, the extension of MF to several layers, inspired by the recent advances in deep learning. We conduct a thorough literature review of multilayer and deep MF, focusing on the models, the choice of the parameters, the applications, and the theoretical aspects. We also illustrate the abilities of deep MF to extract hierarchical features within complex data sets on three showcase examples, namely the extraction of facial features, hyperspectral unmixing (HU) and recommender systems.

We then introduce two new loss functions for deep MF together with a general optimization framework. We show their efficiency to tackle sparse and minVol deep MF on both synthetic and real-world data. These loss functions alleviate the drawbacks of the current approaches both in a theoretical and experimental point of view. We also design a new greedy initialization algorithm for deep MF and extend symmetric NMF to the deep case. We apply this latter successfully to the extraction of overlapping communities of symptoms in psychiatric networks, with promising clinical interpretation.

Finally, we sketch perspectives of future works, including the study of the identifiability of deep MF, the investigation of the connexions between deep MF and deep neural networks, and the exploration of new deep MF models.