

# Exact solutions in classical field theory: Solitons, black holes and boson stars

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Workshop in honor of Prof. Yves Brihaye

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## Talks

**Fabien Buisseret** (CeREF TEchnique & UMONS, Belgium)

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*Z<sub>N</sub>-balls: Existence and application to Yang-Mills plasma*

The equation of state of SU(N) Yang–Mills theory at finite temperature can be modelled by an effective Z<sub>N</sub>-symmetric potential depending on the temperature and on a complex scalar field  $f$ , the Polyakov loop. Allowing  $f$  to be dynamical opens the way to the study of spatially localized classical configurations of the scalar field. We first focus on N=3 and show that spherically symmetric configurations of the scalar field exist in the range  $(1 - 1.2) T_c$ ,  $T_c$  being the deconfinement temperature. The latter configurations can be interpreted as “bubbles” of deconfined gluonic matter; their mean radius is always smaller than 10 fm. We then show that these bubbles can be extended to boson star configurations in curved space, with the same qualitative features and a nonsingular metric. The above results can be found in previous works [1,2]. Finally, we elaborate on the existence of Z<sub>N</sub>-balls for arbitrary N, i.e. localized, finite-energy, solutions of the scalar field with a Z<sub>N</sub>-symmetric potential instead of U(1) as for Q-balls. It is shown that Z<sub>N</sub>-balls may exist in a given potential  $U(f, f^*)$  if  $w = 0$  Q-balls exist in the potential  $U(|f|, |f|)$ . The Z<sub>N</sub>-symmetry does not allow for rotating solutions, neither for radially excited solutions if N is odd. Explicit solutions are built in flat space using a generic potential of polynomial shape.

References:

[1] Y. Brihaye and F. Buisseret, *Gravitating bubbles of gluon plasma above deconfinement temperature*, Symmetry **12**, 1668 (2020) [[arXiv:2009.01038](https://arxiv.org/abs/2009.01038)].

[2] Y. Brihaye and F. Buisseret, *Q-ball formation at the deconfinement temperature in large-Nc QCD*, Phys. Rev. D **87**, 014020 (2013) [[arXiv:1205.1893](https://arxiv.org/abs/1205.1893)].

**Maurizio Consoli** (INFN Catania, Italy)

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*Experimental signals for a second resonance of the Brout-Englert-Higgs field*

It has been recently proposed that, beside the known resonance of mass  $m_h=125$  GeV, the Brout-Englert-Higgs field might exhibit a second, much heavier excitation with a mass of about  $690 \pm 30$  GeV. In spite of its large mass, this hypothetical new state would couple to longitudinal W's with the same typical strength of the lower-mass state and would thus represent a relatively narrow resonance. To compare with experiments, I have analyzed the ATLAS sample of 4-lepton events in the region of invariant mass 620-740 GeV. These data suggest the existence of a new massive resonance of mass  $M_H= 660-680$  GeV and reproduce to high accuracy a characteristic correlation with the lower mass  $m_h=125$  GeV. Altogether, this agreement supports the idea that  $m_h$  and the new  $M_H$  are indeed the masses of two different excitations of the same field.

References:

- M.C., L. Cosmai, Int. J. Mod. Phys. A35 (2020) 2050103; [arXiv:2006.15378 \[hep-ph\]](https://arxiv.org/abs/2006.15378).  
M.C., L. Cosmai, Symmetry 12 (2020) 2037; [doi:10.3390/sym12122037](https://doi.org/10.3390/sym12122037).  
M.C., In Veltman Memorial Volume, Acta Phys. Pol. B52 (2021) 763; [arXiv:2106.06543 \[hep-ph\]](https://arxiv.org/abs/2106.06543).  
M.C., L. Cosmai, [arXiv:2111.08962 \[hep-ph\]](https://arxiv.org/abs/2111.08962), accepted by Int. J. Mod. Phys. A.

**Ludovic Ducobu** (UMONS, Belgium)

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*Hairy black holes, boson stars and non-minimal couplings from Einstein to teleparallel gravity*

Alternative theories of gravity including a scalar field have attracted a lot of attention in the past decade. This is especially true for Horndeski gravity; the most general scalar-tensor theory of gravity in 4D, based on the same geometrical framework as GR, including a real scalar field and presenting second order field equations.

The aim of this talk is to present some of the main results obtained during my PhD thesis under the supervision of Yves Brihaye and related to the topic of spherically symmetric compact objects endowed with scalar fields.

First, I will present some results obtained with Yves in the context of Horndeski gravity. More precisely, I will comment on solutions obtained numerically in a subsector of Horndeski gravity where a scalar field is non-minimally coupled to the Gauss-Bonnet invariant. In particular, I will try to emphasise how the hairy black hole solutions obtained in this context generalised results already known in the literature.

Second, I will introduce recent results of my research in the context of teleparallel theories of gravity. I will start by a short synopsis of the main features of teleparallel theories of gravity compared to metric based theories (especially GR). I will then discuss recent results regarding hairy black holes for some teleparallel theories of gravity endowed with a non-minimally coupled real scalar field.

If time permits, I will comment on how one could possibly extend results obtained for metric based scalar-tensor theories of gravity using the framework of teleparallel gravity.

**Betti Hartmann** (University College London, UK)

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*Black holes and boson stars with new scalar hair*

Static, spherically symmetric black holes can carry scalar hair when coupling standard Einstein gravity minimally to a self-interacting complex scalar field and a U(1) gauge field. For this scalar hair to exist, the frequency of the scalar field needs to be fine-tuned. In this talk, I will discuss these solutions and point out that for sufficiently large gravitational coupling, the space-time splits into two distinct parts: (a) an inflating interior and (b) an exterior which is described by the extremal Reissner-Nordström solution.

Moreover, for a specific range of parameters, the scalar hair develops spatial oscillations, i.e. both black holes as well as boson stars can carry so-called wavy scalar hair.

**Carlos Herdeiro** (Universidade de Aveiro, Portugal)

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*Bosonic stars, gravitational waves and ultralight dark matter*

We discuss, with a concrete theoretical example and real data, how strong gravity systems and observables could assess the nature of dark matter, or at least part of it. In the theoretical modelling, I will focus on fuzzy dark matter and bosonic stars. The real data example will concern the GW190521 gravitational wave event.

**Piotr Kosiński** (University of Lodz, Poland)

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*Chiral particles from conformal symmetry*

I sketch the way the positive energy irreps of conformal group are constructed from the coadjoint orbits. As an example sixdimensional orbits are shown to describe massless particles of definite helicity.

**Jutta Kunz** (Universität Oldenburg, Germany)

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*New Branches of Solutions – From Sphalerons to Black Holes*

After a brief reminiscence of bifurcating branches of sphalerons in Weinberg-Salam theory as well as of black holes within dyons, I will turn to the phenomenon of spontaneous scalarization. While this phenomenon was discovered some time ago in the context of matter induced spontaneous scalarization of neutron stars, much work in recent times has concentrated on curvature induced spontaneous scalarization of black holes, first for static black holes, and later for rotating black holes, where the additional phenomenon of spin induced spontaneous scalarization occurs.

**Eugen Radu** (Universidade de Aveiro, Portugal)

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*Asymptotically Flat, Spherical, Self-Interacting Scalar, Dirac and Proca Stars*

We present a comparative analysis of the self-gravitating solitons that arise in the Einstein–Klein–Gordon, Einstein–Dirac, and Einstein–Proca models, for the particular case of static, spherically symmetric spacetimes. The matter fields possess suitable self-interacting terms in the Lagrangians, which allow for the existence of Q-ball-type solutions in the flat spacetime limit. Our analysis shows the existence of a spin-independent common pattern of the solutions.

*Chern-Simons (CS), Higgs--Chern-Simons (HCS) and Skyrme--Chern-Simons (SCS) densities*

Chern-Simons (CS) densities are derived from Chern-Pontryagin (CP) densities by a one-step dimensional descent. The CP, which are both gauge-invariant and total-divergence, are defined in even dimensions and hence CP are in odd dimensions only. The field equations of CS densities are gauge covariant. The HCP densities result from the dimensional descent over both even and odd dimensions of CP densities, and like the latter are both gauge-invariant and total-divergence. Thus their one-step descent results in HCS densities in all dimensions, endowed with the same properties as the CS. In this spirit, gauge-invariant and total-divergence densities in terms of gauged Skyrme systems can be defined in all dimensions, from which follow the Skyrme-CS densities. Some particular effects of Chern-Simons dynamics are considered, as a motivation for these new actions.