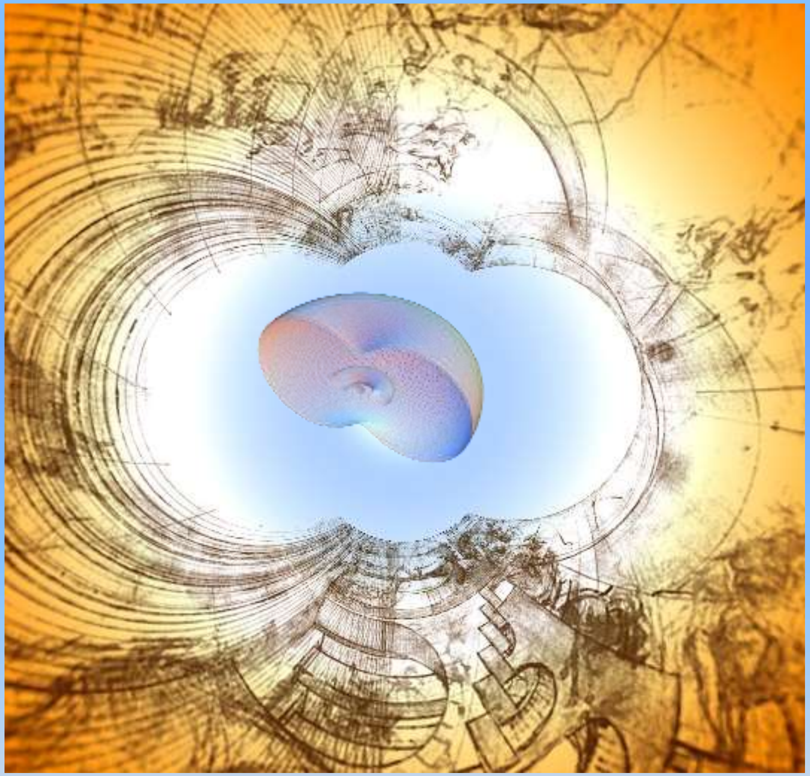
Mons 2.6.2022

Workshop in honour of Yves Brihaye

Asymptotically Flat, Spherical, Self-Interacting Scalar, Dirac and Proca Stars



Eugen Radu

Aveiro University, Portugal

workshop in honour of Professor Yves Brihaye

-***<u>the study of solitons and black holes</u>*: *a central part in Yves' work:***

Yves' first paper

ELSEVIER

Physics Letters B

Volume 66, Issue 4, 14 February 1977, Pages 346-348

Instantons in SU(4) gauge theories

Y. Brihaye, J. Nuyts

***. . .** (decades of work)*

most recent paper:

PHYSICAL REVIEW D

gravitation, and cosmology

Highlights Recent Accepted Collections Authors Referees Staff

Boson stars and black holes with wavy scalar hair

Yves Brihaye and Betti Hartmann

Phys. Rev. D **105**, 104063 – Published 27 May 2022

the framework of this talk:

Gravitating solitons --
scalar, Dirac and Proca stars

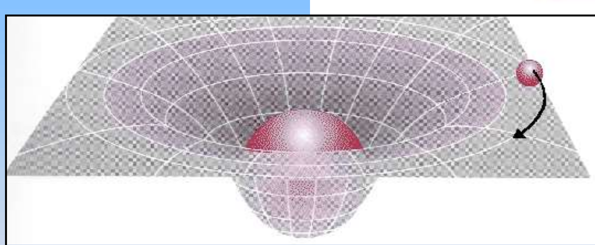
Curvature of space

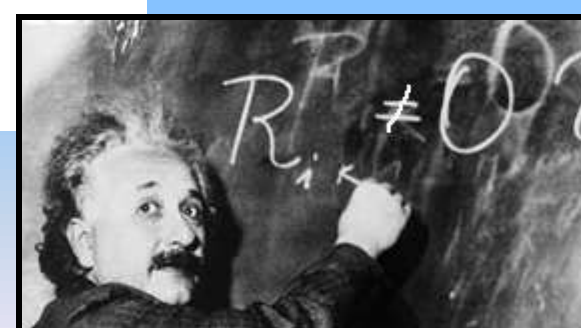
Distribution of mass/energy

$$\boxed{G_{\alpha\beta} = \frac{8\pi G}{c^4} T_{\alpha\beta}}$$

Some constants

fundamental matter fields





our contribution:

symmetry

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Open Access Article

Asymptotically Flat, Spherical, Self-Interacting Scalar, Dirac and Proca Stars

by Carlos A. R. Herdeiro and Eugen Radu *

Departamento de Matemática da Universidade de Aveiro and Centre for Research and Development in Mathematics and Applications (CIDMA), Campus de Santiago, 3810-183 Aveiro, Portugal

* Author to whom correspondence should be addressed.

Symmetry **2020**, *12*(12), 2032; https://doi.org/10.3390/sym12122032

(This article belongs to the Special Issue **Exact Solutions in Classical Field Theory: Solitons, Black Holes and Boson Stars**)

<u>a comparative study of the simmplest static spherically symmetric solitons:</u>

complex, massive fields

$$\left\{ \begin{array}{ll} \textbf{scalar} & s=0 \\ \textbf{Dirac} & s=1/2 \\ \textbf{Proca} & s=1 \end{array} \right\}$$

+ *gravity*

our previous work in this direction:

Physics Letters B
Volume 773, 10 October 2017, Pages 654-662

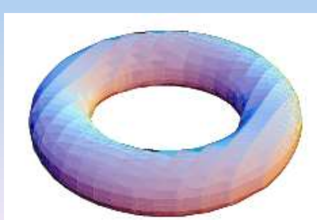
ELSEVIER

Asymptotically flat scalar, Dirac and Proca stars: Discrete *vs.* continuous families of solutions

Carlos A.R. Herdeiro, Alexandre M. Pombo, Eugen Radu

- ***no self-intercation***
- ***spherical symmetry***

- ***no self-intercation***
- ***axially symmetry***



Physics Letters B
Volume 797, 10 October 2019, 134845

ELSEVIER

Asymptotically flat spinning scalar, Dirac and Proca stars

C. Herdeiro [a], I. Perapechka [b], E. Radu [c], Ya. Shnir [d]

s=0 *(scalar field)*

s=1/2 *(Dirac field)*

s=1 *(vector field)*

$$\mathcal{L}_{(0)} = -g^{\alpha\beta}\bar{\Phi}_{,\,\alpha}\Phi_{,\,\beta} - \mu^2\bar{\Phi}\Phi$$

$$\mathcal{L}_{(1/2)} = -i\left[\frac{1}{2}\left(\{\hat{D}\overline{\Psi}\}\Psi - \overline{\Psi}\hat{D}\Psi\right) + \mu\overline{\Psi}\Psi\right]$$

$$\mathcal{L}_{(1)} = -\frac{1}{4}\mathcal{F}_{\alpha\beta}\bar{\mathcal{F}}^{\alpha\beta} - \frac{1}{2}\mu^2\mathcal{A}_\alpha\bar{\mathcal{A}}^\alpha$$

$$\mathcal{U} \to e^{ia}\mathcal{U}$$

spherically symmetric matter ansatz

$$ds^2 = -N(r)\sigma^2(r)dt^2 + \frac{dr^2}{N(r)} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\ , \quad \text{with} \quad N(r) \equiv 1 - \frac{2m(r)}{r}\ .$$

Finite energy, localized solutions: gravitating solitotns

Boson stars

Dirac stars

Proca stars

Kaup (1968)

Ruffin and Bonazzola (1969)

BOSON STARS

PHYSICAL REVIEW VOLUME 172, NUMBER 5 25 AUGUST 1968

Klein-Gordon Geon*

DAVID J. KAUP†

University of Maryland, College Park, Maryland

(Received 4 March 1968)

A study of the spherically symmetric eigenstates of the Klein-Gordon Einstein equations (Klein-Gordon geons) reveals that these geons have properties that are uniquely different from other gravitating systems that have been studied. The equilibrium states of these geons seem analogous to other gravitating systems; but when the question of stability is considered from a thermodynamical viewpoint, it is shown that, in contrast with other systems, adiabatic perturbations are forbidden. The reason is that the equations of state for the thermodynamical variables are not algebraic equations, but instead are differential equations. Consequently, the usual concept of an equation of state breaks down when Klein-Gordon geons are con-

PHYSICAL REVIEW VOLUME 187, NUMBER 5 25 NOVEMBER 1969

Systems of Self-Gravitating Particles in General Relativity and the Concept of an Equation of State*

REMO RUFFINI†

Palmer Physical Laboratory, Princeton University, Princeton, New Jersey 08540

and

Institute for Advanced Study, Princeton, New Jersey 08540

AND

SILVANO BONAZZOLA‡

Facoltà di Matematica, Università di Roma, Roma, Italy

(Received 4 February 1969)

PROCA STARS *(more recently)*

Contents lists available at ScienceDirect

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www.elsevier.com/locate/physletb

ELSEVIER

Proca stars: Gravitating Bose–Einstein condensates of massive spin 1 particles

Richard Brito [a], Vitor Cardoso [a,b], Carlos A.R. Herdeiro [c], Eugen Radu [c]

[a] CENTRA, Departamento de Física, Instituto Superior Técnico – IST, Universidade de Lisboa – UL, Avenida Rovisco Pais 1, 1049, Lisboa, Portugal

[b] Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2J 2W9, Canada

[c] Departamento de Física da Universidade de Aveiro and CIDMA, Campus de Santiago, 3810-183 Aveiro, Portugal

massive spin 1 field

: ***Einstein-Proca theory***

$$\mathcal{S}=\int d^4x\sqrt{-g}\left(\frac{R}{16\pi G}-\frac{1}{4}\mathcal{F}_{\alpha\beta}\bar{\mathcal{F}}^{\alpha\beta}-\frac{1}{2}\mu^2 A_\alpha \bar{A}^\alpha\right)$$

complex massive vector field

A. Proca

DIRAC STARS *(an old problem)*

REVIEWS OF MODERN PHYSICS VOLUME 29, NUMBER 3 JULY, 1957

Interaction of Neutrinos and Gravitational Fields

DIETER R. BRILL AND JOHN A. WHEELER

Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

1. INTRODUCTION; GRAVITATION THE ONLY FORCE IN WHICH NEUTRINOS ARE SUBJECT TO SIMPLE ANALYSIS

KNOWLEDGE of neutrinos to date is confined mainly to emission and absorption processes; that is, to the domain of elementary particle transformations. For comparison, imagine that one knew about electrons only the rate at which they are produced in beta decay, or absorbed in *K*-electron absorption processes, but knew nothing about the motion of electrons in electric and magnetic fields, nothing about the binding of electrons in atoms or the existence of spin-orbit coupling and very little about the stress energy tensor of the

handed polarization that are demanded by the recently gained knowledge.[1–3] Section 4 separates out the radial wave equation for the motion of a neutrino in a centrally symmetric gravitational field, and identifies one term in this equation with a spin-orbit coupling. Section 5 compares and contrasts the energy level spectrum in the case of spherical symmetry for (1) an electron in an electrostatic field, (2) an electron in a gravitational field, (3) a photon in a gravitational field, and (4) a neutrino in a gravitational field. Section 6 recalls the statistical mechanics of an ensemble of neutrinos. Section 7 discusses some neutrino pair creation processes that do not depend upon beta interactions for their existence.

(general formalism but no concrete results)

DIRAC STARS

EVIEW D, VOLUME 59, 104020

Particlelike solutions of the Einstein-Dirac equations

Felix Finster*
Mathematics Department, Harvard University, Cambridge, Massachusetts 02138

Joel Smoller†
Mathematics Department, The University of Michigan, Ann Arbor, Michigan 48109

Shing-Tung Yau‡
Mathematics Department, Harvard University, Cambridge, Massachusetts 02138

(Received 30 January 1998; published 26 April 1999)

The coupled Einstein-Dirac equations for a static, spherically symmetric system of two fermions in a singlet spinor state are derived. Using numerical methods, we construct an infinite number of solitonlike solutions of these equations. The stability of the solutions is analyzed. For weak coupling (i.e., small rest mass of the fermions), all the solutions are linearly stable (with respect to spherically symmetric perturbations), whereas for stronger coupling, both stable and unstable solutions exist. For the physical interpretation, we discuss how the energy of the fermions and the ... ugh gravitation is not renormalizable, ... ved even for strong coupling. [S0556-

The Einstein-Dirac equations take the form

$$R^i_j - \frac{1}{2} R \delta^i_j = -8\pi T^i_j, \quad (\not{D} - m)\Psi = 0, \qquad (1.1)$$

where T^i_j is the energy-momentum tensor of the Dirac particle, $\not{D}$ denotes the Dirac operator (see [13]), and Ψ is the wave function of a fermion of mass m. As in the above-

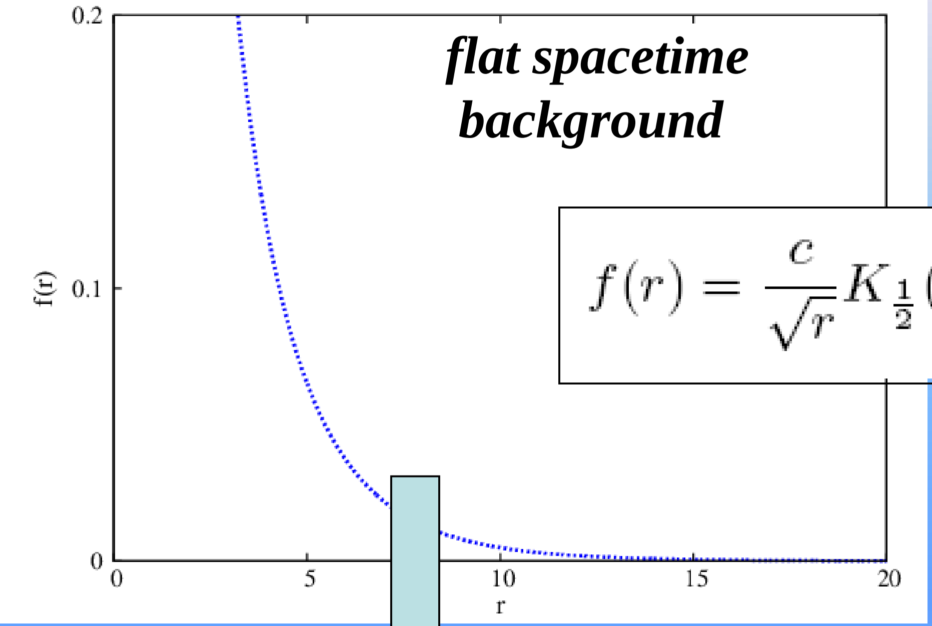
first concrete results

heuristic construction: <u>***gravitational desingularization mechanism***</u>

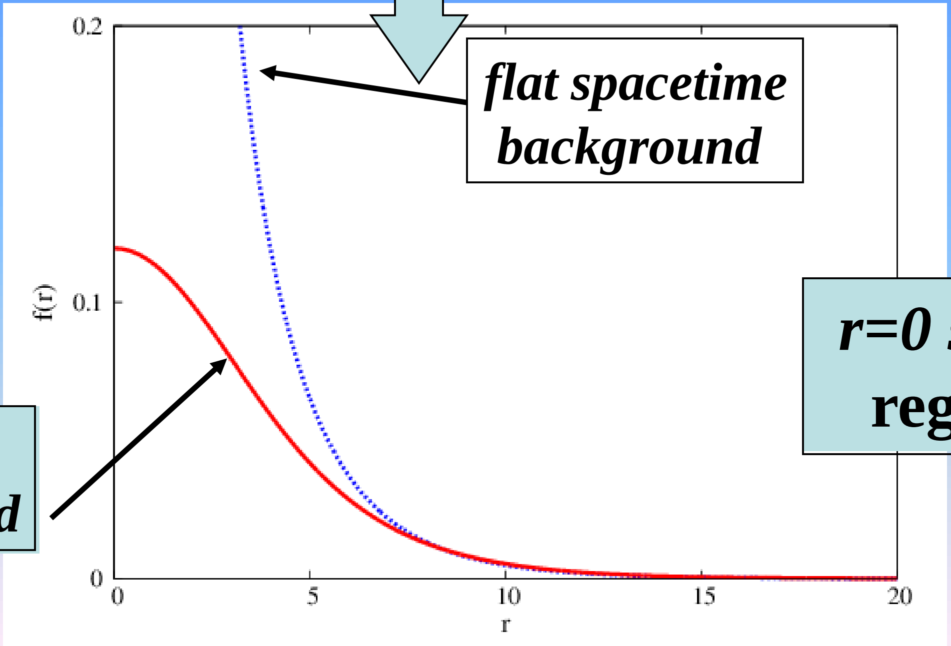
- ***no self-intercation***
- ***spherical symmetry***

e.g. scalar field

$$\Phi = f(r)\, e^{-i\omega t}$$



$$f(r) = \frac{c}{\sqrt{r}} K_{\frac{1}{2}}\left(r\sqrt{\mu^2 - \omega^2}\right)$$



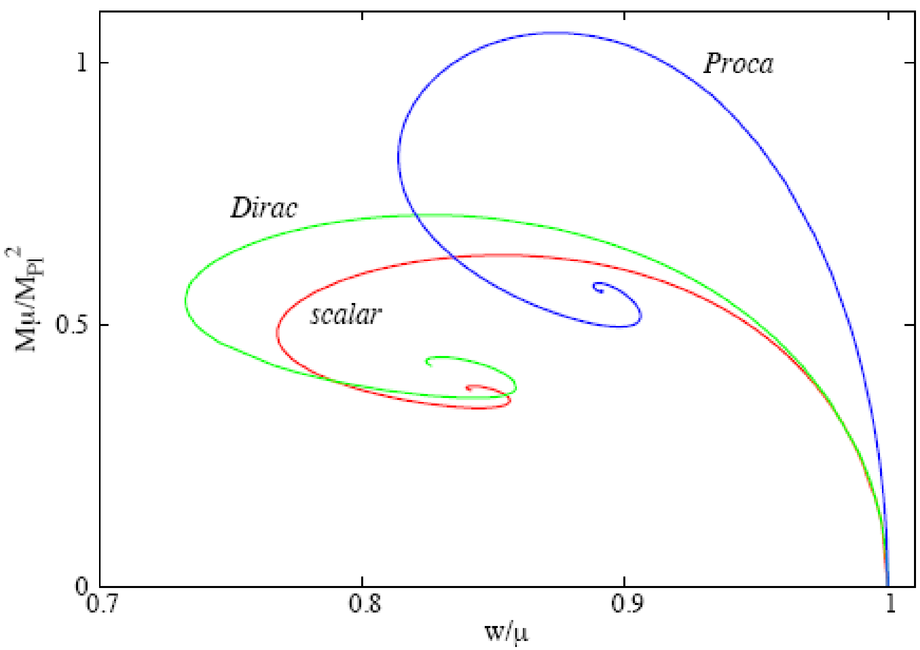
gravity included

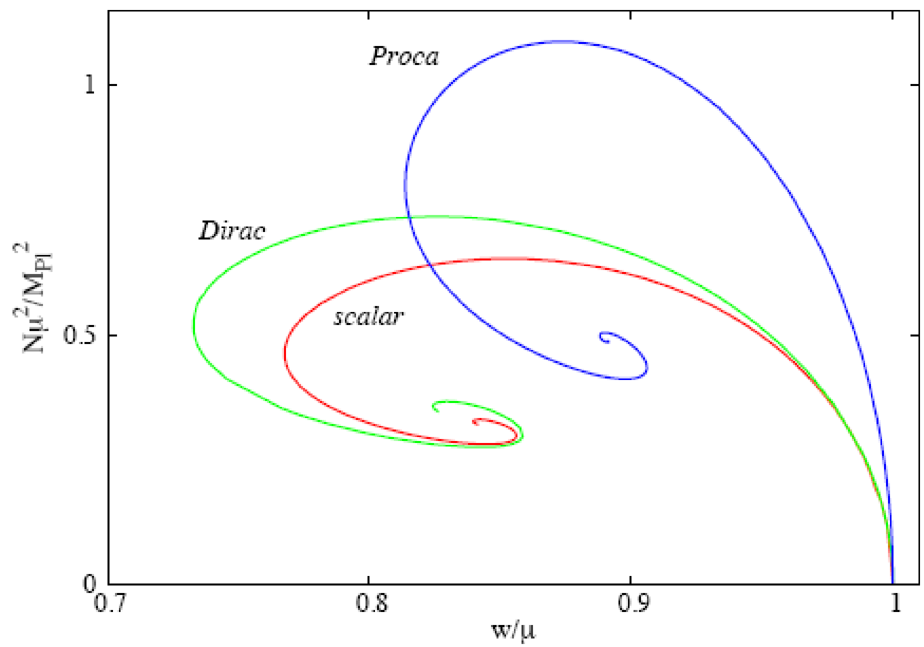
r=0 singularity: regularized

•no self-intercation

arXiv:1708.05674 *(with C. Herdeiro and A. Pombo)*

a comparative study of scalar (s=0), Dirac (s=1/2) and Proca (s=1) stars





•same qualitative picture/ - the basic properties are very similar

however, the bosons and fermions are fundamentally different!

the new results: **adding self-interaction**

$$S = \frac{1}{4\pi}\int d^4x\sqrt{-g}\left[\frac{1}{4G}R - L_{(s)}\right];$$

$$\psi_{(s)} = \{\Phi; \Psi; \mathcal{A}\}$$

- **spin 0:**
  Φ is a complex scalar field,
  $\Phi = \Phi^R + i\Phi^I$.

- **spin 1/2:**
  $\Psi^{(A)}$ are massive spinors, with four complex components.

- **spin 1:**
  $\mathcal{A}$ is a complex 4-potential, with the field strength $\mathcal{F} = d\mathcal{A}$.
  two real vector fields, $\mathcal{A} = \mathcal{A}^R + i\mathcal{A}^I$.

generic action

$$S = \frac{1}{4\pi}\int d^4x\sqrt{-g}\left[\frac{1}{4G}R - L_{(s)}\right]$$

two parts

$$L_{(s)} = L_{(s)}^{(0)} + U_{(s)}^{(\mathrm{int})}$$

$$\text{scalar}: \quad L_{(0)}^{(0)} = g^{\alpha\beta}\Phi^*_{,\alpha}\Phi_{,\beta},$$

$$\text{Dirac}: \quad L_{(1/2)}^{(A)(0)} = i\left[\frac{1}{2}\left(\{\hat{D}\overline{\Psi}^{(A)}\}\Psi^{(A)} - \overline{\Psi}^{(A)}\hat{D}\Psi^{(A)}\right)\right],$$

$$\text{Proca}: \quad L_{(1)}^{(0)} = \frac{1}{4}\mathcal{F}_{\alpha\beta}\bar{\mathcal{F}}^{\alpha\beta}.$$

$$\psi^2 \equiv \{\Phi^*\Phi;\ \mathcal{A}_\alpha\bar{\mathcal{A}}^\alpha;\ i\overline{\Psi}^{(A)}\Psi^{(A)}\}$$

$$\dot{U} \equiv \frac{\partial U}{\partial\psi^2} = \mathcal{M} - 2\lambda\psi^2 + 3\nu\psi^4$$

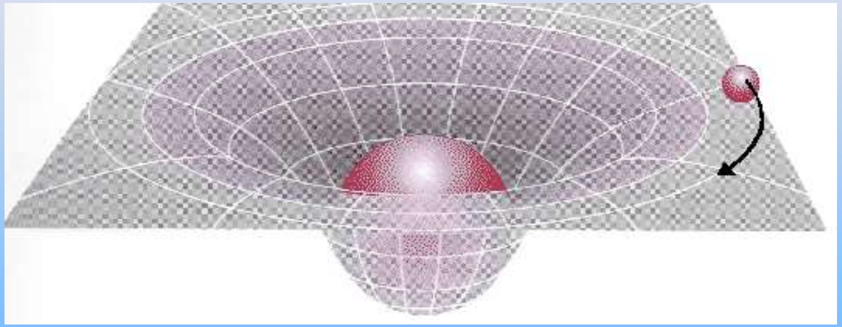
$$U_{(s)}^{(\mathrm{int})} = \mathcal{M}\psi^2 - \lambda\psi^4 + \nu\psi^6 \equiv U$$

Q-ball -type

$$\mathcal{M} \equiv \left\{\mu^2;\ \mu;\ \frac{1}{2}\mu^2\right\}$$

Einstein equattions

$$R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} = 2G\, T_{\alpha\beta}$$



$$\text{scalar}: \quad T_{\alpha\beta} = \Phi^*_{,\alpha}\Phi_{,\beta} + \Phi^*_{,\beta}\Phi_{,\alpha} - g_{\alpha\beta}L_{(0)}$$

$$\text{Dirac}: T_{\alpha\beta} = \sum_A T^{(A)}_{\alpha\beta}, \quad \text{with} \quad T^{(A)}_{\alpha\beta} = -\frac{i}{2}\left[\overline{\Psi}^{(A)}\gamma_{(\alpha}\hat{D}_{\beta)}\Psi^{(A)} - \left\{\hat{D}_{(\alpha}\overline{\Psi}^{(A)}\right\}\gamma_{\beta)}\Psi^{(A)} - g_{\alpha\beta}L_{(1/2)}\right];$$

$$\text{Proca}: T_{\alpha\beta} = \frac{1}{2}(\mathcal{F}_{\alpha\sigma}\bar{\mathcal{F}}_{\beta\gamma} + \bar{\mathcal{F}}_{\alpha\sigma}\mathcal{F}_{\beta\gamma})g^{\sigma\gamma} + \dot{U}(\mathcal{A}_\alpha\bar{\mathcal{A}}_\beta + \bar{\mathcal{A}}_\alpha\mathcal{A}_\beta) - g_{\alpha\beta}L_{(1)}$$

matter fields equattions

$$\text{scalar}: \quad (\nabla^2 - \dot{U})\Phi = 0,$$

$$\text{Dirac}: \quad (\hat{D} - \dot{U})\Psi^{(A)} = 0,$$

$$\text{Proca}: \quad \frac{1}{2}\nabla_\alpha \mathcal{F}^{\alpha\beta} - \dot{U}\mathcal{A}^\beta = 0.$$

four dimensional, asymptotically flat solutions

$$ds^2 = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2).$$

***metric ansatz*:**

$$ds^2 = -N(r)\sigma^2(r)dt^2 + \frac{dr^2}{N(r)} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\ , \quad \text{with} \quad N(r) \equiv 1 - \frac{2m(r)}{r}$$

- ***scalar field***

$$\Phi = \phi(r)e^{-iwt}.$$

the time dependence cancels at the level of the energy momentum tensor, being therefore compatible with a static metric.

- ***Proca field***

$$\mathcal{A} = [F(r)dt + iH(r)dr]\, e^{-iwt}$$

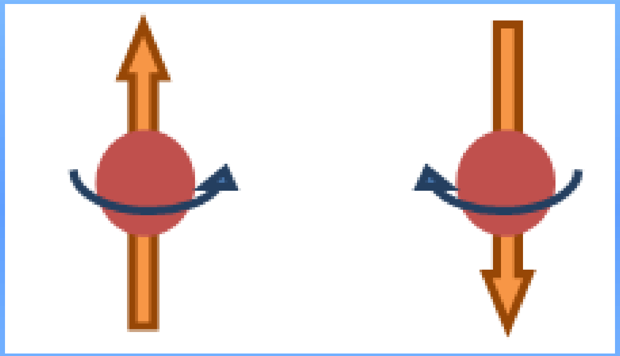
Dirac field

- ***technical problem**: a single spinor field necessarily possesses a nonzero angular momentum – no spherically symmeetric Dirac ansatz*

  - this leads to a difficult numerical problem (PDEs)

- ***solution*** (Finster et. al): ***take (at least) two copies of the field (same mass)***
- ***each component spins in opposite direction:***

  the total angular momentum is zero



- ***then a consistent framework exist for spherically symmetric solutions***
- ***possible generalization with N-copies***

$$\mathcal{L}_D = \sum_{\epsilon=1,2}\left[\frac{\mathrm{i}}{2}\overline{\Psi}_\epsilon\gamma^\nu\hat{D}_\nu\Psi_\epsilon - \frac{\mathrm{i}}{2}\hat{D}_\nu\overline{\Psi}_\epsilon\gamma^\nu\Psi_\epsilon - \mu\overline{\Psi}_\epsilon\Psi_\epsilon\right].$$

standard Dirac Lagrangean

$$\hat{D}_\nu = \partial_\nu - \Gamma_\nu$$

spinor connection matrices etc

two Dirac fields:

$$\mathcal{L}_D = \sum_{\epsilon=1,2}\left[\frac{\mathrm{i}}{2}\overline{\Psi}_{\boldsymbol{\epsilon}}\gamma^\nu \hat{D}_\nu \Psi_{\boldsymbol{\epsilon}} - \frac{\mathrm{i}}{2}\hat{D}_\nu\overline{\Psi}_{\boldsymbol{\epsilon}}\gamma^\nu\Psi_{\boldsymbol{\epsilon}} - \mu\overline{\Psi}_{\boldsymbol{\epsilon}}\Psi_{\boldsymbol{\epsilon}}\right].$$

a spsherically symmetric Ansatz:

$$\Psi_\epsilon = \mathrm{e}^{-\mathrm{i}wt}\mathcal{R}^\epsilon(r)\otimes\Theta^\epsilon(\theta,\varphi)$$

$$\mathcal{R}^{(1)} = -i\mathcal{R}^{(2)} = \begin{bmatrix} z(r) \\ -i\bar{z}(r) \end{bmatrix}$$

$$\Theta^{(1)} = \begin{bmatrix} -\kappa\sin\frac{\theta}{2} \\ \cos\frac{\theta}{2} \end{bmatrix} e^{i\frac{\varphi}{2}}, \quad \Theta^{(2)} = \begin{bmatrix} \kappa\cos\frac{\theta}{2} \\ \sin\frac{\theta}{2} \end{bmatrix} e^{-i\frac{\varphi}{2}}$$

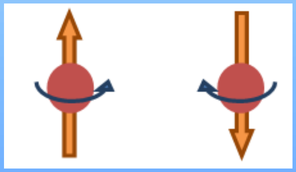
$$\kappa = \pm 1$$

$$z(r) = e^{i\pi/4}f(r) - e^{-i\pi/4}g(r)$$

we follow all conventions and the framework in a series of papers by S. Dolan *functions of r only*

$$T^{r}_{r(1)} = T^{r}_{r(2)},\; T^{r}_{\theta(1)} = T^{\theta}_{\theta(2)} = \; T^{r}_{\varphi(1)} = T^{\varphi}_{\varphi(2)},\;\; T^{r}_{t(1)} = T^{r}_{t(2)}$$

$$T^{t}_{\varphi(1)} = -T^{t}_{\varphi(2)} \quad \Longrightarrow \quad T^{t}_{\varphi} = 0$$



Dirac ansatz is compatible with a spherically symmetric line element

$$ds^2 = -N(r)\sigma^2(r)dt^2 + \frac{dr^2}{N(r)} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\;, \quad \text{with} \;\; N(r) \equiv 1 - \frac{2m(r)}{r}\;.$$

scalar, Dirac, Proca fields: *field equattions*

standard numerical problem: four 1st order ODEs

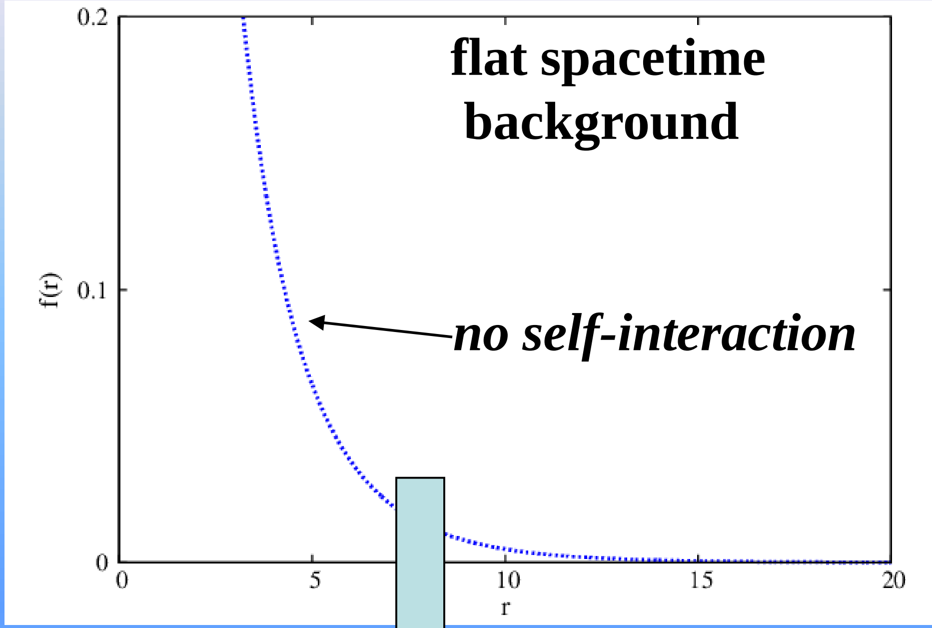
heuristic construction: ***non-linear desingularization mechanism***

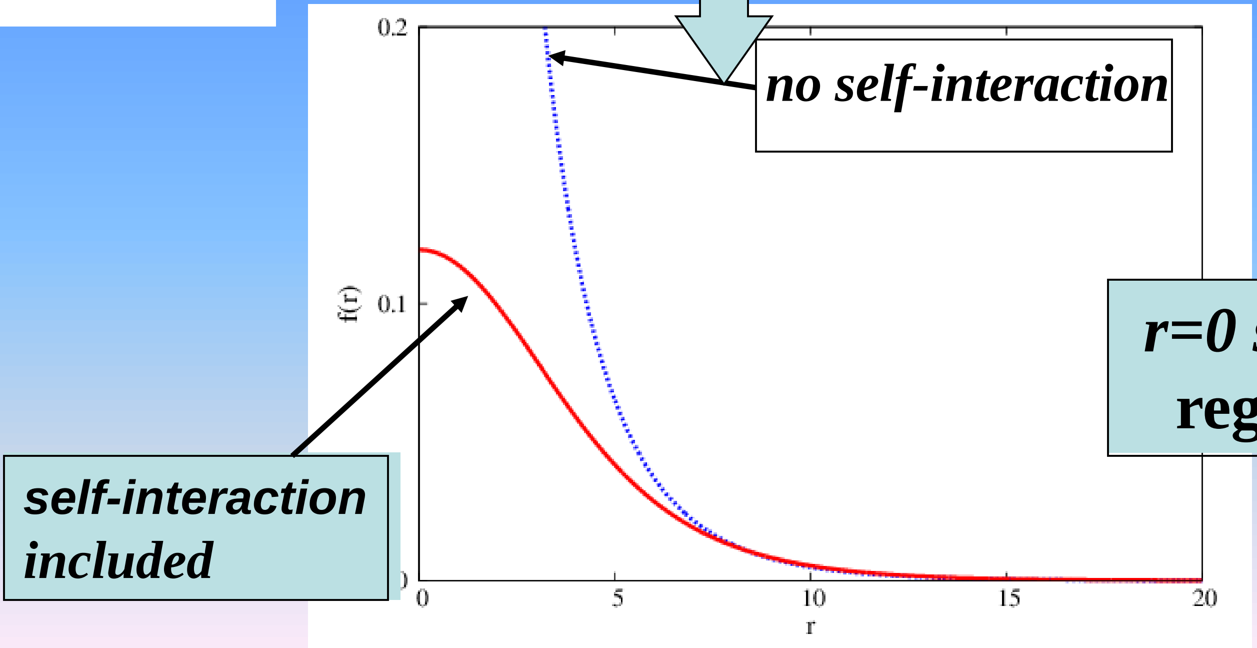
•***works also on flat space***

•***spherical symmetry***

e.g. scalar field

$$\Phi = f(r)\, e^{-i\omega t}$$





best known example

Q-balls

•***the mass of solutions:***

$$\boxed{g_{tt} = -1 + \frac{2M}{r} + \dots}$$

•***the action is invariant under a U(1) global symmetry***:

constant

$$\Psi \to e^{i\alpha}\Psi$$

•***conserved current:***

$$j^a = -i\left(\Psi^* \partial^a \Psi - \Psi \partial^a \Psi^*\right)$$

•Integrating the temporal component of the current on a timelike slice leads to a conserved charge - the ***Noether charge Q:***

$$\boxed{Q = \int_\Sigma j^t}$$

N=Q

•tthe Noether charge counts ***the number of particles***
•this is conserved in the sense of a local continuity equation; **there is no associated Gauss law**!

$$ds^2 = -N(r)\sigma^2(r)dt^2 + \frac{dr^2}{N(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \ ,$$

$$N(r) \equiv 1 - \frac{2m(r)}{r}$$

Einstein equations

$$\text{scalar}: \quad m' = Gr^2\left(N\phi'^2 + \frac{w^2\phi^2}{N\sigma^2} + U\right),$$

$$\text{Dirac}: \quad m' = 2Gr^2\left(4\sqrt{N}(gf' - fg') + \frac{8fg}{r} + U\right),$$

$$\text{Proca}: \quad m' = Gr^2\left[\frac{(F' - wH)^2}{2\sigma^2} + (\mu^2 - 6\lambda\mathcal{A}^2 + 10\nu\mathcal{A}^4)\frac{F^2}{2N\sigma^2} + \frac{U}{\mathcal{A}^2}NG^2\right].$$

$$\text{scalar}: \quad \frac{\sigma'}{\sigma} = 2Gr\left(\phi'^2 + \frac{w^2\phi^2}{N^2\sigma^2}\right),$$

$$\text{Dirac}: \quad \frac{\sigma'}{\sigma} = 8G\frac{r}{\sqrt{N}}\left(gf' - fg' + \frac{w(f^2 + g^2)}{N\sigma}\right),$$

$$\text{Proca}: \quad \frac{\sigma'}{\sigma} = \frac{2Gr}{N}\left(H^2N + \frac{F^2}{N\sigma^2}\right)\dot{U}.$$

matter fields equations

$$\text{scalar}: \quad \phi'' + \left(\frac{2}{r} + \frac{N'}{N} + \frac{\sigma'}{\sigma}\right)\phi' + \frac{w^2}{N^2\sigma^2}\phi - \dot{U}\,\frac{\phi}{N} = 0,$$

$$\text{Dirac}-\text{f}: \quad f' + \left(\frac{N'}{4N} + \frac{\sigma'}{2\sigma} + \frac{1}{r\sqrt{N}} + \frac{1}{r}\right) f - \frac{wg}{N\sigma} + \frac{g}{\sqrt{N}}\dot{U} = 0,$$

$$\text{Dirac}-\text{g}: \quad g' + \left(\frac{N'}{4N} + \frac{\sigma'}{2\sigma} - \frac{1}{r\sqrt{N}} + \frac{1}{r}\right) f + \frac{wf}{N\sigma} + \frac{f}{\sqrt{N}}\dot{U} = 0,$$

$$\text{Proca}-\text{F}: \qquad F' - wH + \frac{2N\sigma^2 H}{w}\dot{U} = 0\ ,$$

$$\text{Proca}-\text{H}: \quad \frac{d}{dr}\left\{\frac{r^2[wH - F']}{\sigma}\right\} + \frac{2r^2 F}{N\sigma}\dot{U} = 0\ .$$

$$\text{scalar}: \quad Q = 2w\int_0^\infty dr\, r^2 \frac{\phi^2}{N\sigma},$$

$$\text{Dirac}: \quad Q = 4\int_0^\infty dr\, r^2 \frac{(f^2+g^2)}{\sqrt{N}},$$

$$\text{Proca}: \quad Q = 2\int_0^\infty dr\, r^2 \frac{(wH - F')N}{\sigma}\ .$$

the issue of self-interaction potential and essential parameters

- make use of scaling properties
- the problems contains **three** essential parameters:

$$\{\alpha, \quad \beta \quad \text{and} \quad w\};$$

$$\Phi = \phi(r)e^{-iwt};$$

Einstein equations

$$R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta} = 2\alpha^2 T_{\alpha\beta}$$

$$\alpha = \frac{M_0}{M_{\rm Pl}}$$

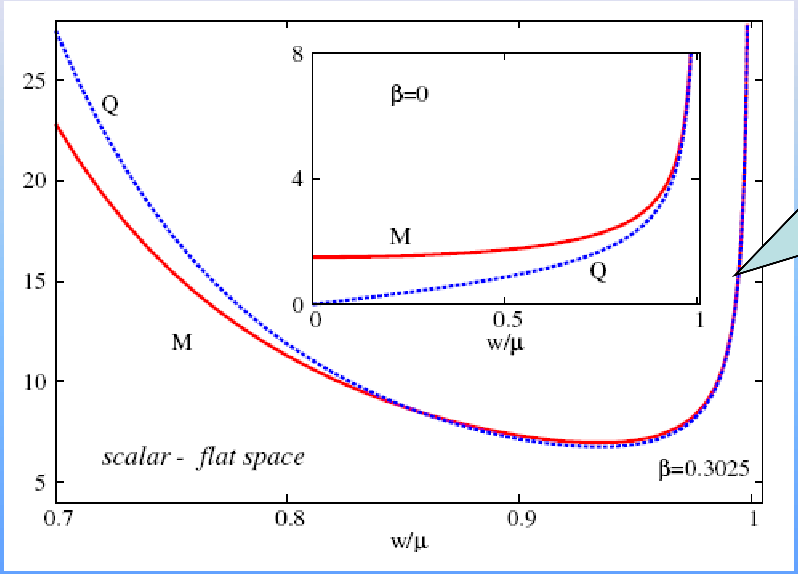
$$\text{scalar and Proca}: \quad M_0 = \frac{\mu}{\sqrt{\lambda}}; \qquad \text{Dirac}: \quad M_0 = \frac{1}{\sqrt{\lambda}}.$$

the potential

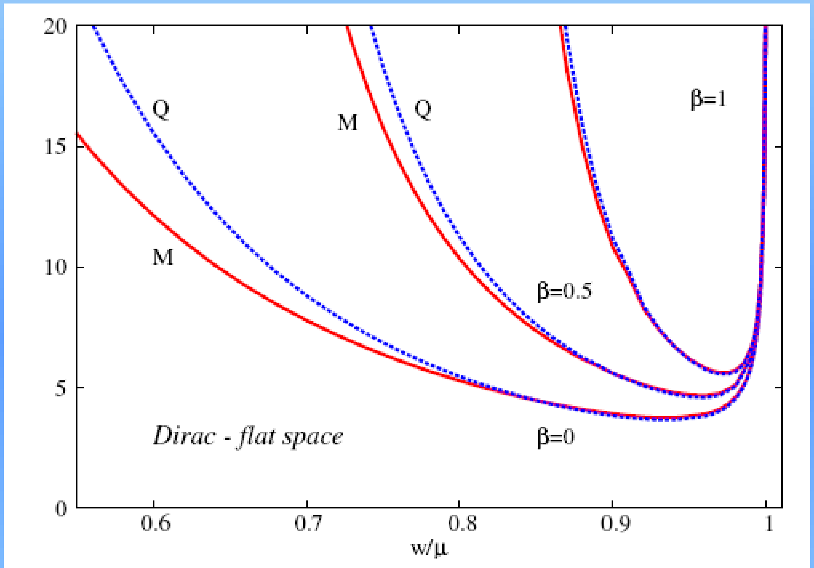
$$U = \psi^2 - \psi^4 + \beta\psi^6$$

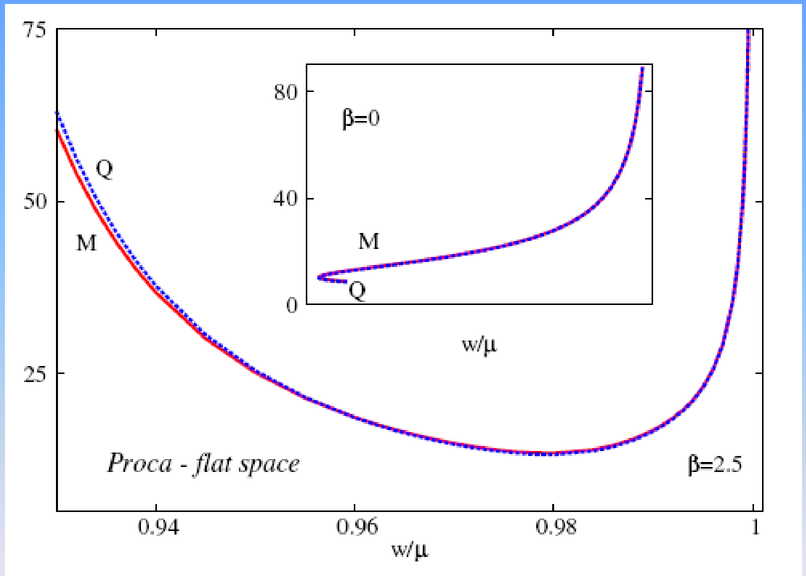
$$\text{scalar and Proca}: \quad \beta = \frac{\nu\mu^2}{\lambda^2}; \qquad \text{Dirac}: \quad \beta = \frac{\nu\mu}{\lambda^2}$$

flat space-time results



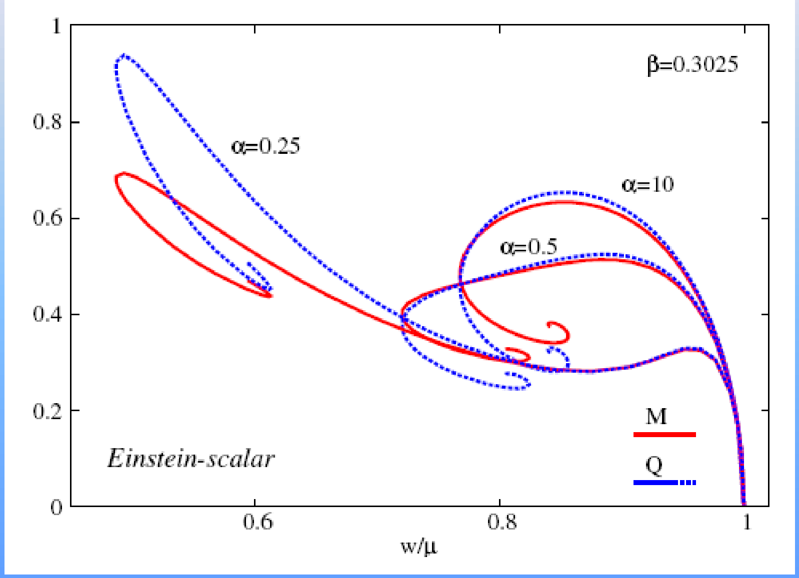
Q-balls

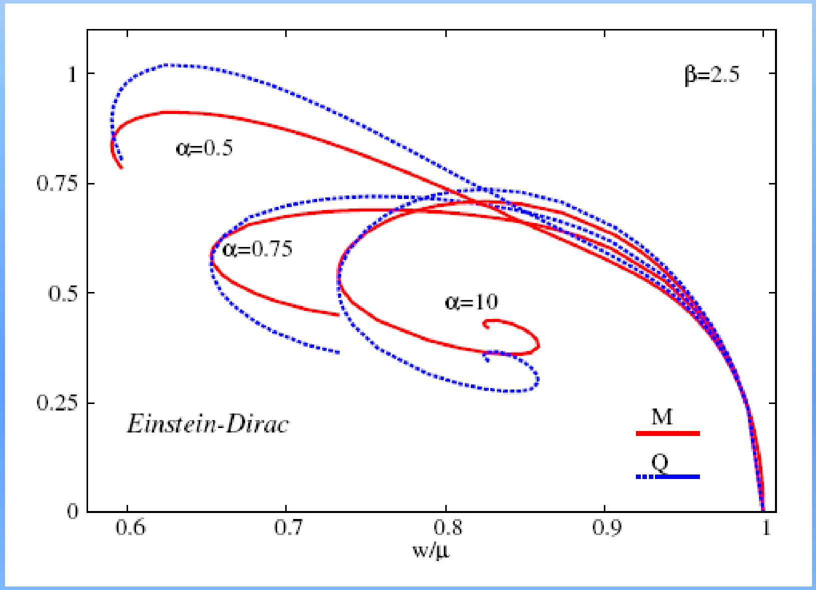


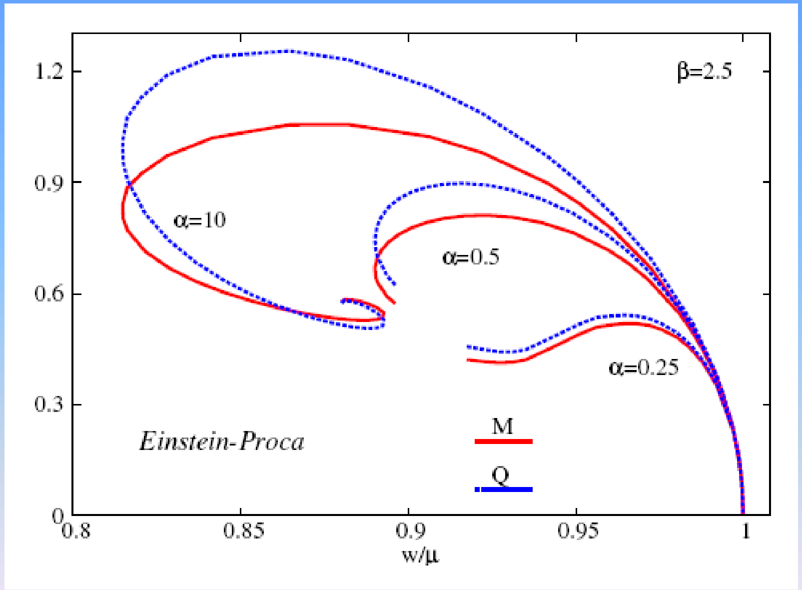


similar pattern

gravitating stars





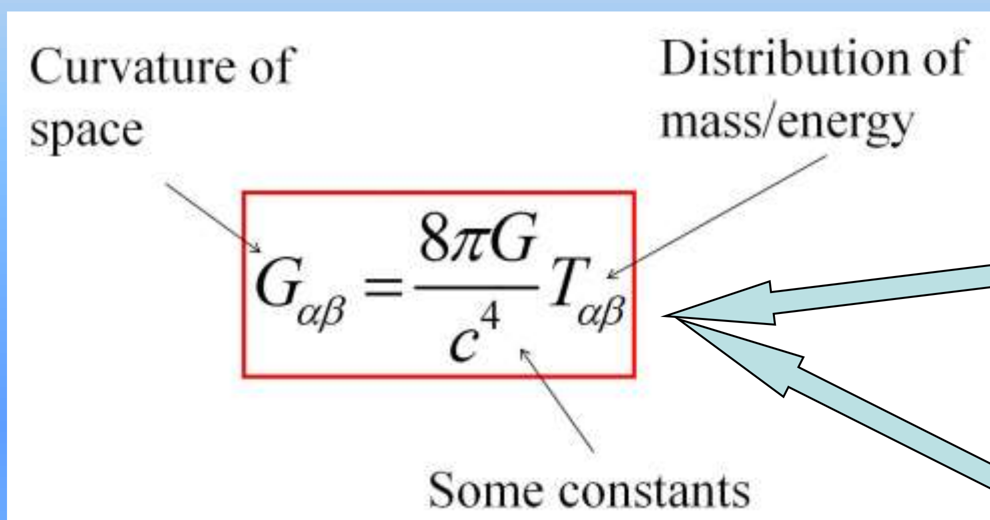


again, similar pattern

certain universality (*bosons and fermions*)

Bosonic vs. fermionic solutions

• *the physical meaning of Einstein equations with fundamental fields*



fundamental matter fields

no 2nd quantization

Q=N=1: mandatory for Dirac case

Pauli exclusion principle

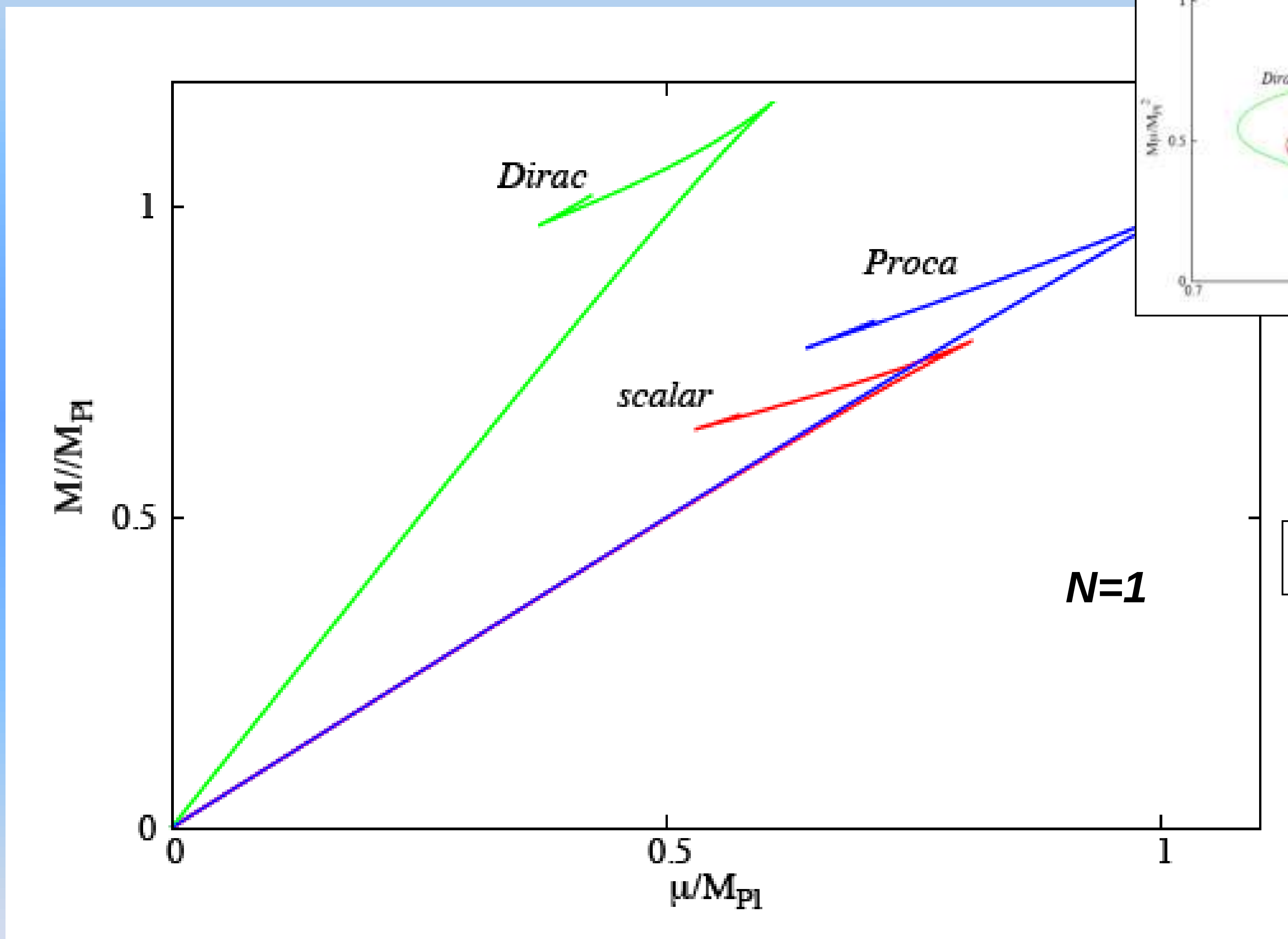
Boson stars: **large number of paricles -- one can ignore quantum effects**
("classic' bosonic fields -- *see* RN BH)

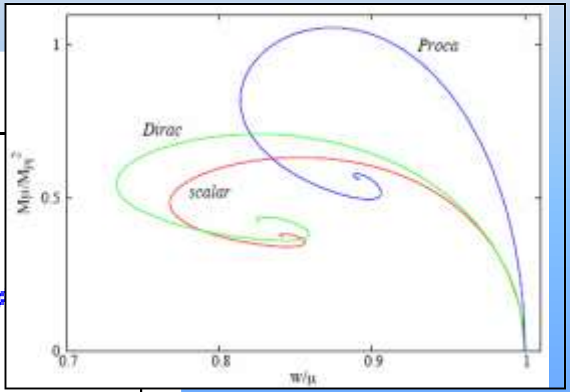
Dirac stars: *unclear* (likely the approach here is inconsistent)

arXiv:1708.05674, 2004.00336 :

• the models possess some scaling symmetry which allows to impose the **N=1** condiition

scale M, μ

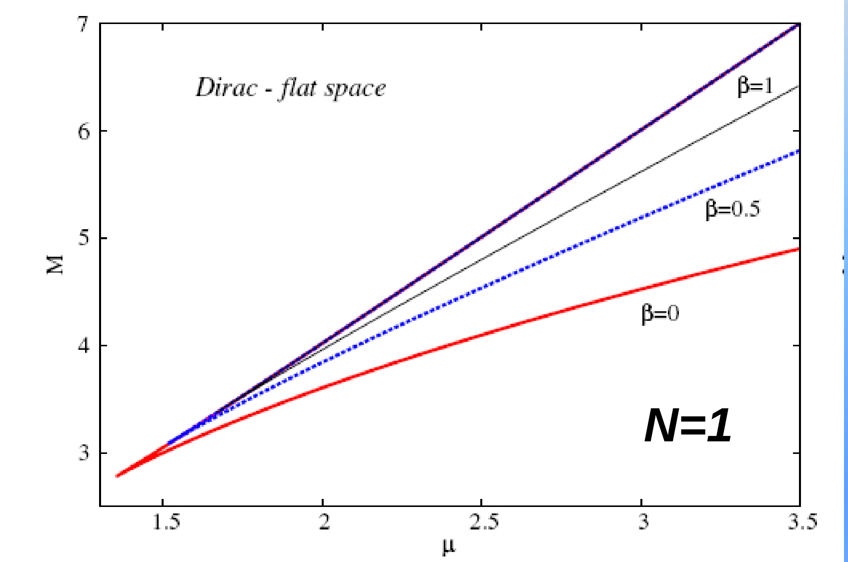


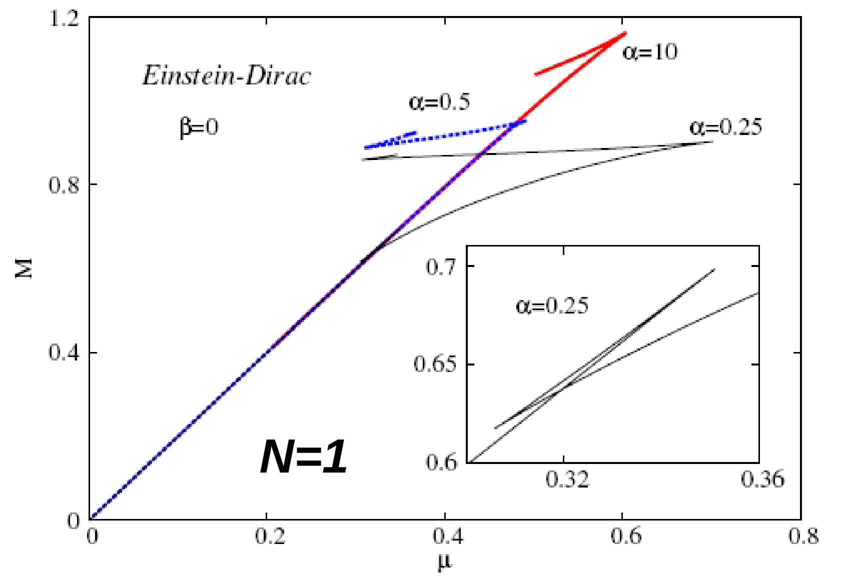


no self-interaction

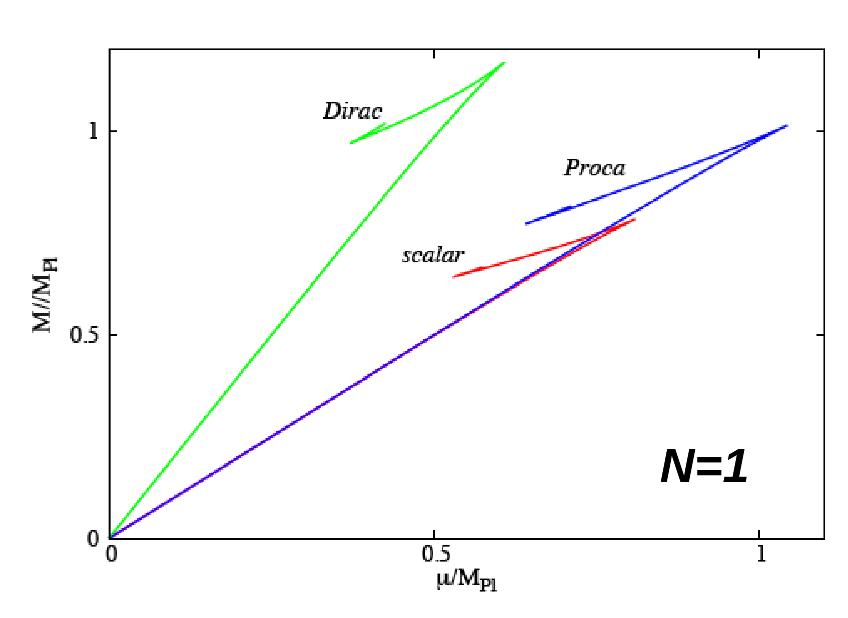
given the field mass, the solution becomes unique -- no longer a continuum scale M, μ

with self-interaction





no self-interaction



Scalar, Dirac and Proca solutions: adding rotation

1906.05386
(with C. Herdeiro, I. Peripechka and Y. Shnir)

- *still no exact solutions*
- *a technically challenging task (PDEs)*

metric

$$ds^2 = -e^{2F_0}dt^2 + e^{2F_1}\left(dr^2 + r^2 d\theta^2\right) + e^{2F_2} r^2 \sin^2\theta \left(d\varphi - \frac{W}{r}dt\right)^2$$

$$\Phi = e^{i(m\varphi - wt)}\phi(r,\theta)$$

scalar

$$\mathcal{A} = e^{i(m\varphi - wt)}\left(iV(r,\theta)dt + \frac{H_1(r,\theta)}{r}dr + H_2(r,\theta)d\theta + iH_3(r,\theta)\sin\theta d\varphi\right)$$

Proca

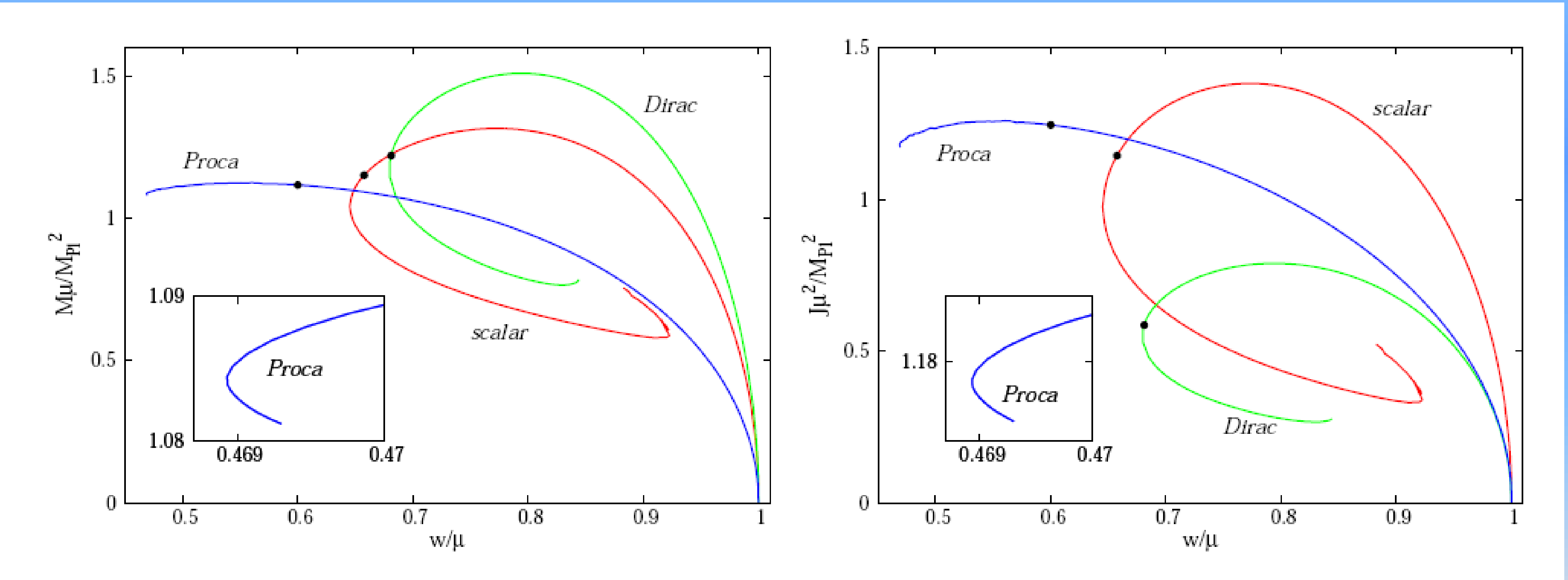
single spinor

$$\Psi = e^{i(m\varphi - wt)}\begin{pmatrix}\psi_1(r,\theta)\\ \psi_2(r,\theta)\\ -i\psi_1^*(r,\theta)\\ -i\psi_2^*(r,\theta)\end{pmatrix}, \quad \text{with} \quad \psi_1(r,\theta) = P(r,\theta) + iQ(r,\theta), \quad \psi_2(r,\theta) = X(r,\theta) + iY(r,\theta).$$

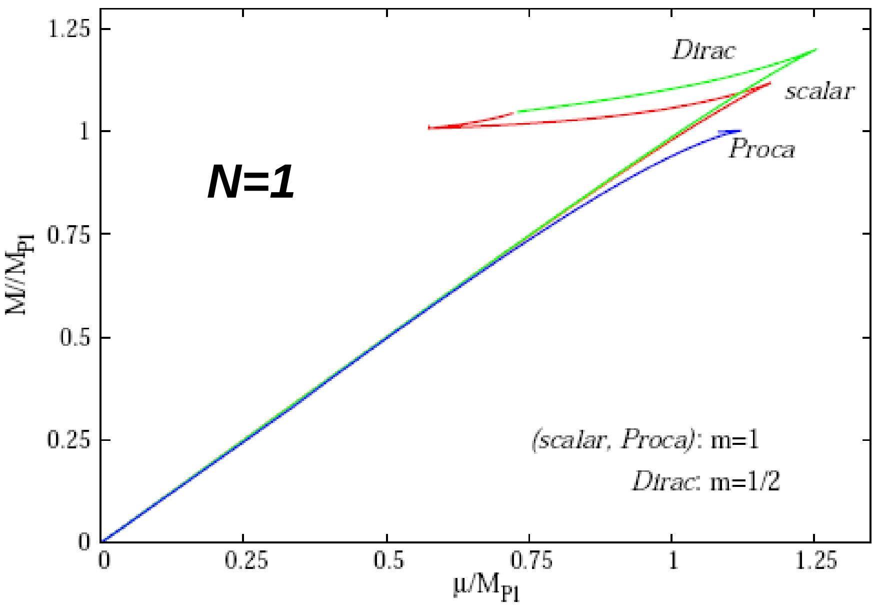
general result: $\boxed{J = mQ}$

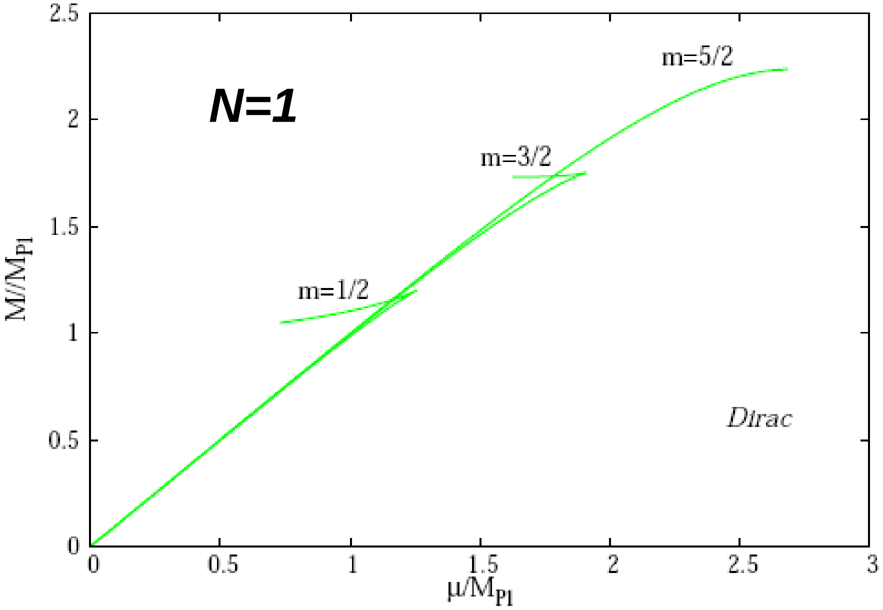
1906.05386
(with C. Herdeiro, I. Peripechka and Y. Shnir)

- *same qualitative picture as with spherically symmetric solutions*
- *all conceptial remarks from there still holds*



• *results after imposing the one-particle condition*





to summarize:

•***Einstein equations with matter field(s) sources:***

- *a variety of interesting solutions*

•***Topologically trivial, solitionic configurations***

(non-perturbative effects)

Yves' work

…many other

•***Boson (s=0,1) and Dirac Stars:* common basic features**

•***Q-ball – type solutions* in flat space supported by self-interaction**

- same pattern with gravity

Open issues to study in the future

• ***Stability? Dynamical properties? Astrophysics?***

(Carlos' talk tomorrow)

•***<u>Einstein-Dirac system:</u> the physical meaning?***

(still an <u>interesting mathematical problem)</u>

-***Black Holes with Dirac hair?***

$$\boxed{w \sim \Omega_H}$$

- so far known for bosons only

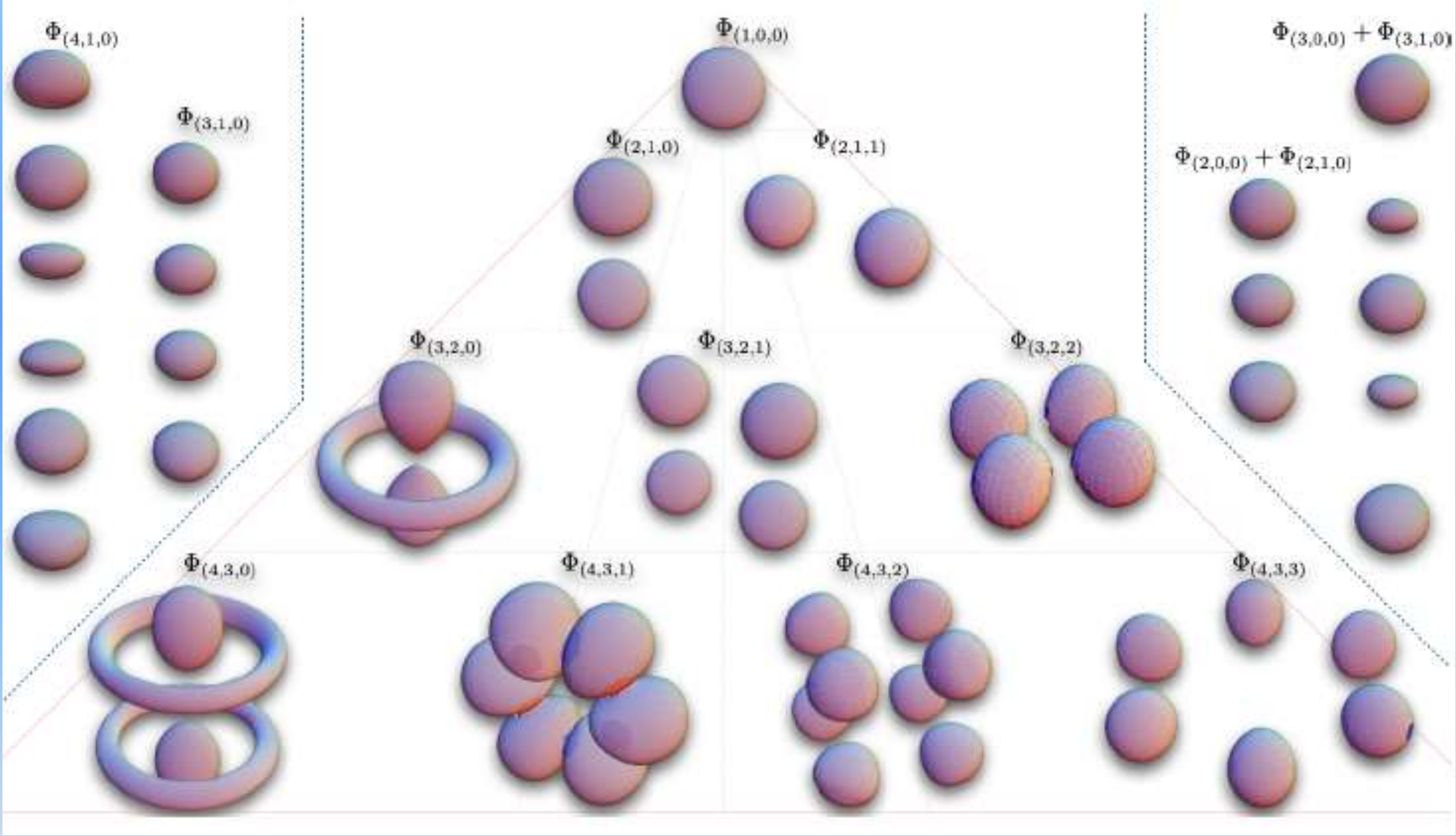
• ***synchronization mechanism***

- ***not enough for a Dirac field***

-*more general solutions?*

multipolar boson stars: e-Print: 2008.10608
)(with C. Herdeiro, J. Kunz, I. Perapechka, and Y. Shnir)

scalar field
s=0



- similar solutions should exist for **s=1/2, s=1**

self-interacting Proca field
recent result:

real field

$$\mathcal{L} = \frac{1}{16\pi} R - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - V\left(X^2\right)$$

The problem with Proca: ghost instabilities in self-interacting vector fields

Katy Clough (Queen Mary, U. of London, Math. Sci.), Thomas Helfer (Johns Hopkins U.), Helvi Witek (Illinois U., Urbana), Emanuele Berti (Johns Hopkins U.)

Apr 22, 2022

10 pages

e-Print: 2204.10868 [gr-qc]

View in: ADS Abstract Service

$$F_{\mu\nu} = \nabla_\mu X_\nu - \nabla_\nu X_\mu$$

$$V' = dV/d(X^2)$$

The intrinsic pathology of self-interacting vector fields

Andrew Coates (Koc U.), Fethi M. Ramazanoğlu (Koc U.)

May 16, 2022

4 pages

e-Print: 2205.07784 [gr-qc]

View in: ADS Abstract Service

__effective metric__

$$\hat{g}_{\mu\nu} = \frac{2}{\mu^2}\left[V' g_{\mu\nu} + 2V'' X_\mu X_\nu\right]$$

Proca wave equation $\hat{g}_{\rho\sigma} \nabla^\rho \nabla^\sigma X_\mu - \mathcal{M}_{\mu\rho} X^\rho = 0$

$\hat{g}_{nn} = 0$

- the time evolution is mathematically ill-defined
- **__no problem for a free Proca field__**

Happy birthday Yves!!

..and hope to meet us soon in Mons