Carrollian Perspective on Celestial Holography

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References

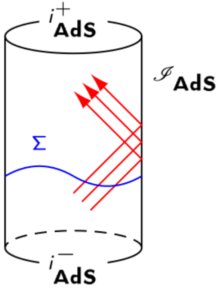
- Based on:

1. Carrollian Perspective on Celestial Holography
   Laura Donnay, Adrien Fiorucci, Yannick Herfray, Romain Ruzziconi
   `arXiv:2202.04702` Phys.Rev.Lett. (2022)

2. Flat Space Holography: from Null Infinity to the Celestial Sphere
   Laura Donnay, Adrien Fiorucci, Yannick Herfray, Romain Ruzziconi
   `arXiv:22xx.xxxxx`

Motivations

- Holographic principle:
  Gravity in a given spacetime region can be encoded on a lower-dimensional boundary of that region.
- Explicit realization of this principle: AdS/CFT correspondence
- Bottom-up approach: use what we know from gravity in the bulk to construct a dual theory
  $\Longrightarrow$ Very efficient thanks to the powerful constraints implied by the conformal symmetries at the boundary
- Interesting properties of AdS/CFT:
  1. Asymptotic symmetries in the bulk = global symmetries in the dual theory
     $\Longrightarrow$ The dual theory is a CFT living on the timelike boundary
  2. "Gravity in a box" (implemented by Dirichlet boundary conditions)
     $\Longrightarrow$ Closed system
     $\Longrightarrow$ Charges are conserved



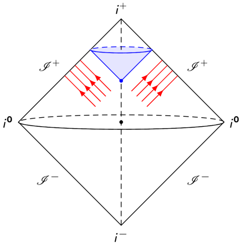
- How general is the holographic principle? Does it extend to asymptotically flat spacetimes?

Flat space holography program

(see *e.g.* [Susskind '99] [Polchinski '99] [Giddings '00] [de Boer-Solodukhin '03] [Arcioni-Dappiaggi '03] [Mann-Marolf '06] for early attempts).

- Asymptotic symmetries form the Bondi-van der Burg-Metzner-Sachs (BMS) group [Bondi-van der Burg-Metzner '62] [Sachs '62]

  $\Longrightarrow$ Broadly studied in the literature (see *e.g.* [Newman-Unti '62] [Penrose '65] [Geroch '77] [Ashtekar-Streubel '81] [Barnich-Troessaert '10])



- Important obstructions to flat space holography:
  1. Null nature of $\mathscr{I}^+$ and $\mathscr{I}^-$
  2. Radiation leaking through the conformal boundary
     $\Longrightarrow$ Open gravitational system
     $\Longrightarrow$ The BMS charges are not conserved

- How to construct the holographic dual?

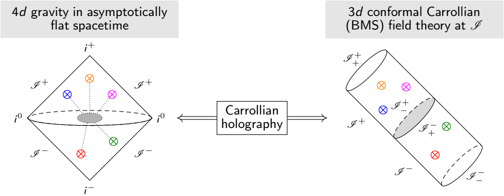
Holographic nature of null infinity

- *Two distinct but complementary visions of $\mathscr{I}^+$:*

Picture 1: Carrollian holography	**Picture 2: Celestial holography**
$\mathscr{I}^+$ is seen as a boundary along which there is an evolution with respect to u	$\mathscr{I}^+$ is seen as a portion of Cauchy hypersurface pushed to infinity
Describe the dynamics of the system	Describe the state of the system
Flux-balance laws	Scattering problem between $\mathscr{I}^-$ and $\mathscr{I}^+$
Suggests a $4d$ bulk / $3d$ boundary *Carrollian holography*	Suggests a $4d$ bulk / $2d$ boundary *celestial holography*
Dual: $3d$ BMS field theory	Dual: $2d$ Celestial CFT

Carrollian Holography

- *Carrollian holography*:



- BMS algebra $\simeq$ conformal Carrollian algebra [Duval-Gibbons-Horvathy '14].
  $\Longrightarrow$ Dual theory: Carrollian CFT in $3d$.
- "Carroll" refers to the $c \to 0$ limit of the Poincaré group [Lévy-Leblond '65].
  $\Longrightarrow$ Carrollian physics naturally induced on null hypersurfaces.
- Carrollian holography follows a similar pattern than AdS/CFT correspondence: $4d$ bulk / $3d$ boundary duality.
  $\Longrightarrow$ Naturally arises from a flat limit procedure ($\Lambda \to 0$).
  $\Longrightarrow$ The flat limit in the bulk induces a Carrollian limit at the boundary.

Pros *vs* Cons of Carrollian holography

- Success of this approach:
  1. 3d gravity (*e.g.* entropy matching, entanglement entropy, effective action, correlation functions, Carroll anomaly ...).
     [Barnich-Gomberoff-Gonzalez '12] [Barnich '12] [Bagchi-Detournay-Fareghbal-Simon '13] [Bagchi-Fareghbal '12] [Detournay-Grumiller-Scholler-Simon '14] [Bagchi-Basu-Grumiller-Riegler '15] [Hartong '16] [Bagchi-Grumiller-Merbis '16] [Campoleoni-Ciambelli-Delfante-Marteau-Petropoulos-Ruzziconi '22]
  2. Fluid/gravity correspondence:
     [Ciambelli-Marteau-Petkou-Petropoulos-Siampos '18]

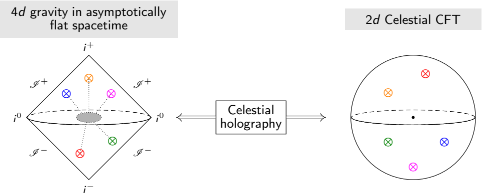
$$\boxed{\text{Gravity in asymptotically flat spacetime}} \iff \boxed{\text{Carrollian fluid at the boundary}}$$

- Drawbacks:
  1. Few is known about quantum Carrollian CFTs.
  2. How to treat the non-conservation of the charges generated by outgoing radiation?

$\Longrightarrow$ One of the goals of this talk is to address these 2 issues.

Celestial holography

- *Celestial holography:*



- $\mathcal{S}$-matrix elements in the bulk $\Longleftrightarrow$ Correlation functions in a 2d CFT [see Laura's talk]
  [de Boer-Solodukhin '03] [Pasterski-Shao-Strominger '17] [Strominger '18] [Donnay-Puhm-Strominger '19] [Fotopoulos-Taylor '19]
- Massless scattering $\Longrightarrow$ Mellin transform: [de Boer-Solodukhin '03] [Pasterski-Shao-Strominger '17] [Pasterski-Shao '17]

$$\langle \mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\dots\mathcal{O}_{\Delta_N,J_N}(z_N,\bar{z}_N)\rangle = \left(\prod_{i=1}^{N}\int_0^{+\infty} d\omega_i\,\omega_i^{\Delta_i-1}\right)\mathcal{A}(\{\omega_i\},\{z_i\},\{\bar{z}_i\})$$

  where the CCFT operators $\mathcal{O}_{\Delta_i,J_i}(z_i,\bar{z}_i)$ are characterized by conformal dimension Δ_i and spin J_i.
- Conformal symmetries in CCFT induced by Lorentz transformations in the bulk:
  $\mathrm{Conf}(S^2) \simeq$ Lorentz

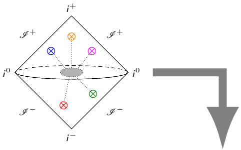
Pros *vs* Cons of celestial holography

- Advantages and successes of this approach:
  1. Use the powerful techniques of CFT (OPEs in CCFT are obtained by collinear limit of bulk amplitudes).
  2. Ward identities in the CCFT encode the soft theorems in the bulk.
  3. New $w_{1+\infty}$ symmetries uncovered in the CCFT OPEs.
     $\Longrightarrow$ Infinite tower of soft theorems in the bulk.
     $\Longrightarrow$ Provides an organization of the solution space in gravity.

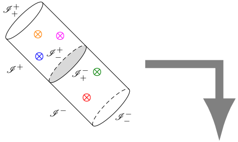
     [Strominger '21] [Guevara-Himwich-Pate-Strominger '21] [Adamo-Mason-Sharma '21] [Freidel-Pranzetti-Raclariu '21] [Compère-Oliveri-Seraj '22] [Bu-Heuveline-Skinner '22]
- Drawbacks:
  1. No clear path to relate with AdS_4/CFT_3.
  2. Information on the dynamics not manifest.
     (*Where is time in celestial holography? How to encode the flux-balance laws?*).

$\Longrightarrow$ One of the goals of this talk is to address these 2 issues.

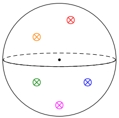
Objectives



- **From the bulk...** Gravity in 4d asymptotically flat spacetime:
  - $\Longrightarrow$ Bondi gauge.
  - $\Longrightarrow$ BMS and conformal Carroll symmetries.
  - $\Longrightarrow$ Surface charges and flux-balance laws.

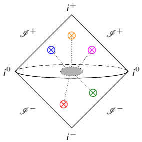


- **...to null infinity...** Carrollian CFT at null infinity:
  - $\Longrightarrow$ Describe the non-conservation at null infinity in terms of external sources.
  - $\Longrightarrow$ Write the Ward identities of a sourced Carrollian CFT.
  - $\Longrightarrow$ Show that the Ward identities holographically describe the asymptotic bulk dynamics.

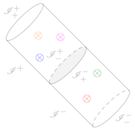


- **...to the celestial sphere.** Relate the Carrollian and the celestial approaches:
  - $\Longrightarrow$ Relate Carrollian and CCFT operators through an appropriate integral transform.
  - $\Longrightarrow$ Demonstrate the equivalence between the Ward identities of the sourced Carrollian CFT and those of the CCFT.

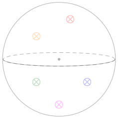
From the bulk...



...to null infinity...



...to the celestial sphere.

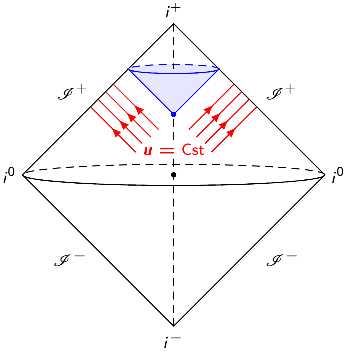


Solution space of $4d$ asymptotically flat spacetimes

- Asymptotically flat metric in Bondi coordinates to study $\mathscr{I}^+$: (u, r, x^A) where $x^A = (z, \bar{z})$ [Bondi-van der Burg-Metzner '62] [Sachs '62]:

$$ds^2 = \left(\frac{2M}{r} + \mathcal{O}(r^{-2})\right) du^2 - 2\left(1 + \mathcal{O}(r^{-2})\right) dudr + \left(r^2 \mathring{q}_{AB} + r\, C_{AB} + \mathcal{O}(r^0)\right) dx^A dx^B + \left(\frac{1}{2}\partial_B C^B_A + \frac{2}{3r}(N_A + \frac{1}{4} C^B_A \partial_C C^C_B) + \mathcal{O}(r^{-2})\right) dudx^A \, .$$

- Flat boundary metric: $\mathring{q}_{AB} dx^A dx^B = 2 dz d\bar{z}$.
- Minkowski metric: $ds^2_{\text{Mink}} = -2dudr + 2r^2 dz d\bar{z}$.
- Subleading corrections in r with respect to Minkowski metric are obtained by solving the Einstein equations. They involve functions of (u, x^A):
  1. C_{AB} : asymptotic shear,
  2. $N_{AB} = \partial_u C_{AB}$: Bondi news (outgoing radiation),
  3. M : mass aspect,
  4. N_A : angular momentum aspect.



- *Time evolution/constraint equations* on the mass and angular momentum aspects

$$\partial_u M = -\frac{1}{8} N_{AB} N^{AB} + \frac{1}{4} \partial_A \partial_B N^{AB},$$
$$\partial_u N_A = \partial_A M + \frac{1}{16} \partial_A (N_{BC} C^{BC}) - \frac{1}{4} N^{BC} \partial_A C_{BC} - \frac{1}{4} \partial_B (C^{BC} N_{AC} - N^{BC} C_{AC}) - \frac{1}{4} \partial_B \partial^B \partial^C C_{AC} + \frac{1}{4} \partial_B \partial_A \partial_C C^{BC},$$

with $N_{AB} = \partial_u C_{AB}$ the Bondi news tensor.

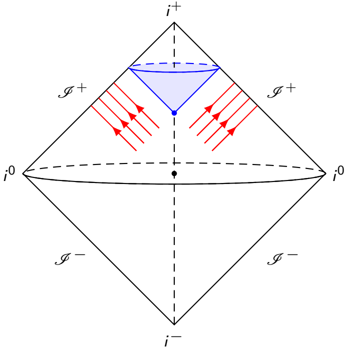
- *Bondi mass loss formula*:

$$\boxed{\partial_u \left[\int_{S^2_\infty} d^2 z\, M \right] = -\frac{1}{8} \int_{S^2_\infty} d^2 z\, N_{AB} N^{AB} \leq 0.}$$

$\implies$ The mass decreases in time due to the emission of gravitational waves.

$\implies$ Important argument to show the existence of gravitational waves at a non-linear level of the theory.

$\implies$ The analysis at $\mathscr{I}^+$ provides some information on the dynamics of the system.



Asymptotic symmetries

- Diffeomorphisms preserving the solution space: $\xi = \xi^u \partial_u + \xi^z \partial + \xi^{\bar{z}} \bar{\partial} + \xi^r \partial_r$ with

$$\xi^u = \mathcal{T} + \frac{u}{2}(\partial \mathcal{Y} + \bar{\partial} \bar{\mathcal{Y}}), \quad \xi^z = \mathcal{Y} + \mathcal{O}(r^{-1}), \quad \xi^{\bar{z}} = \bar{\mathcal{Y}} + \mathcal{O}(r^{-1}), \quad \xi^r = -\frac{r}{2}(\partial \mathcal{Y} + \bar{\partial} \bar{\mathcal{Y}}) + \mathcal{O}(r^0),$$

  where

  1. $\mathcal{T} = \mathcal{T}(z, \bar{z})$ is the supertranslation parameter;
  2. $\mathcal{Y} = \mathcal{Y}(z)$, $\bar{\mathcal{Y}} = \bar{\mathcal{Y}}(\bar{z})$ are the superrotation parameters satisfying the conformal Killing equation.

- Using the modified Lie bracket $[\xi_1, \xi_2]_\star = [\xi_1, \xi_2] - \delta_{\xi_1} \xi_2 + \delta_{\xi_2} \xi_1$ that takes into account the field-dependence of the vector fields [Barnich-Troessaert '10]:

$$[\xi(\mathcal{T}_1, \mathcal{Y}_1, \bar{\mathcal{Y}}_1), \xi(\mathcal{T}_2, \mathcal{Y}_2, \bar{\mathcal{Y}}_2)]_\star = \xi(\mathcal{T}_{12}, \mathcal{Y}_{12}, \bar{\mathcal{Y}}_{12}),$$

  with

$$\mathcal{T}_{12} = \mathcal{Y}_1 \partial \mathcal{T}_2 - \frac{1}{2} \partial \mathcal{Y}_1 \mathcal{T}_2 - (1 \leftrightarrow 2) + \text{c.c.}, \quad \mathcal{Y}_{12} = \mathcal{Y}_1 \partial \mathcal{Y}_2 - (1 \leftrightarrow 2), \quad \bar{\mathcal{Y}}_{12} = \bar{\mathcal{Y}}_1 \bar{\partial} \bar{\mathcal{Y}}_2 - (1 \leftrightarrow 2)$$

  where c.c. stands for complex conjugate terms. $\Longrightarrow$ $\boxed{\mathfrak{bms}_4 \text{ algebra.}}$

  (Extended BMS: $\mathfrak{bms}_4^{\text{ext}} = (\text{Witt} \oplus \text{Witt}) \oplus\!\!\!\!\!\!\;\,\in \mathfrak{supertranslations}^*$ [Barnich-Troessaert '10])

  $\Longrightarrow$ Extended BMS algebra is the most convenient asymptotic symmetry algebra for holographic discussions.
  $\Longrightarrow$ See [Schwarz '22] for a recent discussion favouring Virasoro instead of Diff($\mathcal{S}^2$) superrotations.

Conformal Carroll $\simeq$ BMS

- Carrollian structure on $\mathscr{I}^+$ with coordinates $x^a = (u, z, \bar{z})$ [Geroch '77]:

$$(q_{ab}, n^c) \quad \text{with} \quad q_{ab} n^b = 0 \,.$$

- Consistently with the Bondi metric, $q_{ab} dx^a dx^b = 0 du^2 + 2 dz d\bar{z}$ and $n^a \partial_a = \partial_u$.
- Conformal Carrollian symmetries are generated by vector fields $\bar{\xi} = \bar{\xi}^a \partial_a$ on $\mathscr{I}^+$ satisfying

$$\mathcal{L}_{\bar{\xi}} q_{ab} = 2\alpha q_{ab} \,, \qquad \mathcal{L}_{\bar{\xi}} n^a = -\alpha n^a \,,$$

  with $\alpha = \frac{1}{2}(\partial \mathcal{Y} + \bar{\partial} \bar{\mathcal{Y}})$.
- Solution:

$$\bar{\xi} = \left[\mathcal{T} + \frac{u}{2}(\partial \mathcal{Y} + \bar{\partial} \bar{\mathcal{Y}}) \right] \partial_u + \mathcal{Y} \partial + \bar{\mathcal{Y}} \bar{\partial} \,.$$

$\Longrightarrow$ Coincides with the restriction on $\mathscr{I}^+$ of the bulk BMS asymptotic Killing vectors.
$\Longrightarrow$ The standard Lie bracket of these vector fields reproduces the $\mathfrak{bms}_4$ algebra.

- Isomorphism: $\boxed{\mathfrak{bms}_4 \simeq \text{Conformal Carroll algebra.}}$ [Duval-Gibbons-Horvathy '14]

BMS surface charges

- Poisson structure on the radiative phase space [Ashtekar-Streubel '81]:
$$\{N_{zz}(u,z,\bar{z}), C_{\bar{w}\bar{w}}(u',w,\bar{w})\} = 16\pi G\, \delta(u-u')\, \delta^{(2)}(z-w)\,.$$
- At a cut $\mathcal{S}_u \equiv \{u = \text{constant}\}$ of $\mathscr{I}^+$, one can construct "surface charges" associated with BMS symmetries using covariant phase space methods [Wald-Zoupas '99] [Barnich-Troessaert '10].
- BMS charges are non-integrable and non-conserved due to the outgoing radiation at $\mathscr{I}^+$.
$\Longrightarrow$ Typical properties for a dissipative system.
- *Selection of a meaningful integrable part:*
[Compère-Fiorucci-Ruzziconi '18] [Campiglia-Peraza '20] [Compère-Fiorucci-Ruzziconi '20] [Donnay-Ruzziconi '21] [Freidel-Pranzetti '21]:
$$\bar{H}_\xi[g] = \frac{1}{8\pi G}\int_{\mathcal{S}_u} d^2 z\, [2\mathcal{T}\mathcal{M} + \mathcal{Y}\bar{\mathcal{N}} + \bar{\mathcal{Y}}\mathcal{N}]$$
with
$$\mathcal{M} = M + \frac{1}{8}(C_{zz}N^{zz} + C_{\bar{z}\bar{z}}N^{\bar{z}\bar{z}}), \qquad \mathcal{N} = N_{\bar{z}} - u\bar{\partial}\mathcal{M} + \frac{1}{4}C_{\bar{z}\bar{z}}\bar{\partial}C^{\bar{z}\bar{z}} + \frac{3}{16}\bar{\partial}(C_{zz}C^{zz})\Big]$$
$$+ \frac{u}{4}\bar{\partial}\Big[\Big(\partial^2 - \frac{1}{2}N_{zz}\Big)C^z_{\bar{z}} - \Big(\bar{\partial}^2 - \frac{1}{2}N_{\bar{z}\bar{z}}\Big)C^{\bar{z}}_z\Big].$$
- Remark: $\mathcal{M} = -\text{Re}\Psi^0_2$, $\mathcal{N} = -\Psi^0_1 + u\eth\Psi^0_2$.
- Using the Barnich-Troessaert bracket [Barnich-Troessaert '10] or one of its refinements [Freidel-Oliveri-Pranzetti-Speziale '21], one can write charge algebra for "open gravitational system", from which one can deduce the BMS flux-balance laws:
$$\frac{d}{du}\bar{H}_\xi[g] = \mathcal{F}_\xi[g] \neq 0.$$

BMS fluxes

- BMS fluxes [see Laura's talk]:

$$\frac{d\bar{H}_\xi}{du} = \int_{\mathcal{S}_u} dz d\bar{z} \, (F^H_\xi + F^S_\xi) \, ,$$

$$F^H_\xi = \frac{1}{16\pi G}\left[-\frac{1}{2}\mathcal{T} N^{zz} N_{zz} - \frac{u}{2} N^{zz} N_{zz} \partial \mathcal{Y} + \frac{1}{4} \mathcal{Y} \partial (C_{zz} N^{zz}) + \frac{1}{2} \mathcal{Y} C_{zz} \partial N^{zz} \right] + \text{c.c.} \, ,$$

$$F^S_\xi = \frac{1}{16\pi G} N^{zz} \left(\partial^2 \mathcal{T} + u \, \partial^3 \mathcal{Y} \right) + \text{c.c.} \, .$$

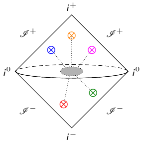
- F^H_ξ: hard flux (quadratic in C_{AB} and N_{AB}), F^S_ξ: soft flux (linear in C_{AB} and N_{AB}).
- Properties:
  1. The BMS charges are conserved when $N_{AB} = 0$.
  2. The BMS fluxes generate canonically the transformations on the radiative phase space: [He-Lysov-Mitra-Strominger '14] [Kapec-Lysov-Pasterski-Strominger '14]

$$\left\{ \int_{\mathscr{I}^+} du dz d\bar{z} \, F^{S,H}_\xi (u, z, \bar{z}), C_{AB}(u', w, \bar{w}) \right\} = -\delta^{S,H}_\xi C_{AB}(u', w, \bar{w}).$$

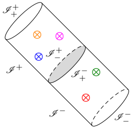
  3. The BMS fluxes form a representation of the BMS algebra: $\{ \int_{\mathscr{I}^+} F_{\xi_1}, \int_{\mathscr{I}^+} F_{\xi_2} \} = - \int_{\mathscr{I}^+} F_{[\xi_1, \xi_2]}$ for the standard bracket $\{ \int_{\mathscr{I}^+} F_{\xi_1}, \int_{\mathscr{I}^+} F_{\xi_2} \} = \int_{\mathscr{I}^+} \delta_{\xi_1} F_{\xi_2}$. [Campiglia-Peraza '20] [Compère-Fiorucci-Ruzziconi '20] [Donnay-Ruzziconi '21]

- Remark: a similar analysis can be performed in the advanced Bondi coordinates (v, r, x^A) at $\mathscr{I}^-$.

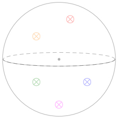
From the bulk...



...to null infinity...



...to the celestial sphere.



Sourced Ward identities

- Goal: Establish a framework that holographically encodes the leaks through the conformal boundary.

$\Longrightarrow$ Coupling with external sources [See Adrien's talk].

- Consider a QFT on a manifold $\mathscr{M}$ with coordinates x^a.
- Fields: $\Phi^i(x)$, symmetries: $\delta_K \Phi^i = K^i[\Phi]$, conserved Noether currents: $\partial_a j^a_K(x) = 0$.
- Couple the theory with external sources $\sigma(x)$:

$\Longrightarrow$ Classically, generically breaks the Noetherian symmetries;
$\Longrightarrow$ Noether currents are no longer conserved [Troessaert '15] [Barnich-Fiorucci-Ruzziconi, *to appear*]:

$$\partial_a j^a_K(x) = F_K(x), \qquad F_K(x)|_{\sigma=0} = 0\,.$$

- At the quantum level, sourced Ward identities (key result) [Donnay-Herfray-Fiorucci-Ruzziconi '22]:

$$\boxed{\langle \partial_a j^a_K(x) X\rangle + \frac{\hbar}{i}\sum_{i=1}^{N} \delta^{(n)}(x - x_i)\, \delta_{K^i} \langle X\rangle = \langle F_K(x) X\rangle}$$

with

1. $X \equiv \Phi^{i_1}(x_1)\dots\Phi^{i_N}(x_N)$: insertions of operators;
2. $\delta_{K^i}\langle X\rangle \equiv \langle \Phi^{i_1}(x_1)\dots K^i[\Phi(x_i)]\dots\Phi^{i_N}(x_N)\rangle$.

$\Longrightarrow$ With no field insertion: $\langle \partial_a j^a_K(x)\rangle = \langle F_K(x)\rangle$ (reproduces the classical equation);
$\Longrightarrow$ In absence of sources: $\langle \partial_a j^a_K(x) X\rangle + \frac{\hbar}{i}\sum_{i=1}^{N} \delta^{(n)}(x - x_i)\, \delta_{K^i}\langle X\rangle = 0$ (standard result).

- Integrated Ward identity:

$$\boxed{\sum_{i=1}^{N} \delta_{K^i} \langle X \rangle = \frac{i}{\hbar} \left\langle \left(\int_{\mathscr{M}} \mathbf{F}_K - \int_{\partial \mathscr{M}} \mathbf{j}_K \right) X \right\rangle}$$

with $\mathbf{F}_K = F_K \, (d^n x)$ and $\mathbf{j}_K = j^a_K \, (d^{n-1}x)_a$.

- Usual textbook result recovered after assuming:
  1. No external sources $\implies \mathbf{F}_K = 0$;
  2. Noether currents vanish at the boundary $\implies \mathbf{j}_K|_{\partial \mathscr{M}} = 0$.

$$\sum_{i=1}^{N} \delta_{K^i} \langle X \rangle = 0 \,.$$

$\implies$ Invariance of the correlators.

- For our purpose, we will not make these assumptions.

Application to Carrollian CFT

- Consider a $3d$ Carrollian CFT. Local coordinates: $x^a = (u, z, \bar{z})$. Carrollian structure: $ds^2 = 0\, du^2 + 2dzd\bar{z}$ and $n^a\partial_a = \partial_u$.
- Noether currents:

$$j^a_{\bar{\xi}} = \mathcal{C}^a{}_b \bar{\xi}^b, \qquad \mathcal{C}^a{}_b = \begin{bmatrix} \mathcal{M} & \mathcal{N}_{\mathcal{B}} \\ \mathcal{B}^A & \mathcal{A}^A{}_B \end{bmatrix}.$$

$\Longrightarrow$ $\mathcal{C}^a{}_b$: Carrollian stress-tensor;
$\Longrightarrow$ $\mathcal{M}$, $\mathcal{N}_{\mathcal{B}}$, $\mathcal{B}^A$, $\mathcal{A}^A{}_B$: Carrollian momenta.
[Ciambelli-Marteau-Petkou-Petropoulos-Siampos '18] [de Boer, Hartong, Obers, Sybesma, Vandoren '18] [Ciambelli-Marteau '18] [Donnay-Marteau '19] [Chandrasekaran-Flanagan-Shehzad- Speranza '21] [Freidel-Pranzetti '21]

- Noether currents associated with $3d$ global conformal Carrollian symmetries ($\simeq 4d$ Poincaré symmetries) satisfy the classical flux-balance law $\partial_a j^a_{\bar{\xi}}(x) = F_{\bar{\xi}}(x)$, with $F_{\bar{\xi}} = F_a \bar{\xi}^a$, provided

$$\begin{array}{rcccl} \textit{Carrollian translations} & : & \partial_b & \Rightarrow & \partial_a \mathcal{C}^a{}_b = F_b\,, \\ \textit{Carrollian rotation} & : & -z\partial + \bar{z}\bar{\partial} & \Rightarrow & \mathcal{C}^z{}_z - \mathcal{C}^{\bar{z}}{}_{\bar{z}} = 0\,, \\ \textit{Carrollian boosts} & : & \bar{x}^A \partial_u & \Rightarrow & \mathcal{C}^A{}_u = 0\,, \\ \textit{Carrollian dilatation} & : & x^a \partial_a & \Rightarrow & \mathcal{C}^a{}_a = 0\,, \end{array}$$

- No further constraints coming from supertranslations and superrotations.
- Constraints on the Carrollian momenta:

$$\begin{aligned} \partial_u \mathcal{M} &= F_u\,, & \mathcal{B}^A &= 0\,, \\ \partial_u \mathcal{N}_z - \tfrac{1}{2}\partial\mathcal{M} + \bar{\partial}\mathcal{A}^{\bar{z}}{}_z &= F_z\,, & 2\mathcal{A}^z{}_z + \mathcal{M} &= 0\,, \\ \partial_u \mathcal{N}_{\bar{z}} - \tfrac{1}{2}\bar{\partial}\mathcal{M} + \partial\mathcal{A}^z{}_{\bar{z}} &= F_{\bar{z}}\,, & 2\mathcal{A}^{\bar{z}}{}_{\bar{z}} + \mathcal{M} &= 0\,. \end{aligned}$$

Sourced Conformal Carrollian Ward identities

- Consider the sourced Ward identities with (quasi) conformal Carrollian primary fields insertions:

$$\delta_{\xi}\Phi_{(k,\bar{k})} = \left[\left(\mathcal{T} + \frac{u}{2}(\partial\mathcal{Y} + \bar{\partial}\bar{\mathcal{Y}})\right)\partial_u + \mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial} + k\,\partial\mathcal{Y} + \bar{k}\,\bar{\partial}\bar{\mathcal{Y}}\right]\Phi_{(k,\bar{k})}\,.$$

- Carrollian weights $(k, \bar{k})$ are integers or half-integers.
- In terms of the Carrollian momenta:

$$\partial_u \langle \mathcal{M}\, X\rangle + \frac{\hbar}{i}\sum_i \delta^{(3)}(x - x_i)\partial_{u_i}\langle X\rangle = \langle F_u\, X\rangle \;,$$

$$\partial_u \langle \mathcal{N}_z\, X\rangle - \frac{1}{2}\partial\langle \mathcal{M}\, X\rangle + \bar{\partial}\langle \mathcal{A}^{\bar{z}}{}_z\, X\rangle + \frac{\hbar}{i}\sum_i \left[\delta^{(3)}(x - x_i)\partial_i\langle X\rangle - \partial\left(\delta^{(3)}(x - x_i)\, k_i\,\langle X\rangle\right)\right] = \langle F_z\, X\rangle \;,$$

$$\langle \mathcal{B}^A\, X\rangle = 0\,,$$

$$\langle(\mathcal{A}^z{}_z + \frac{1}{2}\mathcal{M})X\rangle + \frac{\hbar}{i}\sum_i \delta^{(3)}(x - x_i)\, k_i\;\langle X\rangle = 0\,.$$

(together with the complex conjugate relations)

- Remark: with no field insertion, the sourced Ward identities reproduce the VEV of the classical relations.
- Claim:

The sourced Ward identities holographically encode the asymptotic dynamics of gravity in asymptotically flat spacetimes.

Holographic correspondence

- Correspondence between boundary Carrollian momenta and bulk gravitational data at $\mathscr{I}^+$ [Donnay-Herfray-Fiorucci-Ruzziconi '22]:

$$\langle \mathcal{M} \rangle = \frac{1}{4\pi G}\left[M + \frac{1}{8}(C_{AB}N^{AB})\right] , \quad \langle \mathcal{A}^A{}_B \rangle = -\frac{1}{2}\delta^A{}_B \langle \mathcal{M} \rangle ,$$

$$\langle \mathcal{N}_A \rangle = \frac{1}{8\pi G}\left(N_A + \frac{1}{4}C_A^B \partial_C C_B^C + \frac{3}{32}\partial_A (C_B^C C_C^B) + \frac{u}{4}\partial^B(\partial_B\partial_C - \frac{1}{2}N_{BC})C_A^C - \frac{u}{4}\partial^B(\partial_A\partial_C - \frac{1}{2}N_{AC})C_B^C \right) .$$

- Fixed by requiring compatibility between boundary Noether currents and bulk gravitational charges.
- Similar to the AdS/CFT dictionary where the holographic stress-energy tensor of the CFT is identified with some subleading order in the expansion of the bulk metric. [Balasubramanian-Kraus '99] [de Haro-Solodukhin-Skenderis '01]
- External sources identified with the news: $\sigma_{AB} = N_{AB}$.
- Dissipation through the fluxes:

$$F_u = -\frac{1}{16\pi G}\left[\sigma^{zz}\sigma_{zz} - 2(\bar{\partial}^2\sigma_{zz} + \partial^2\sigma_{\bar{z}\bar{z}})\right] ,$$

$$F_z = \frac{1}{32\pi G}\left[\partial(\sigma^{zz}\Phi_{zz}) + 2\Phi_{zz}\partial\sigma^{zz} + u\partial(\bar{\partial}^2\sigma_{zz} - \partial^2\sigma_{\bar{z}\bar{z}})\right] , \quad F_{\bar{z}} = \bar{F}_z .$$

- $\Phi_{zz} \equiv \Phi_{(\frac{3}{2},-\frac{1}{2})}$ holographically associated with the asymptotic shear: $\langle \Phi_{AB} \rangle = C_{AB}$, (Compatibility: $\langle \partial_u \Phi_{AB} \rangle = \sigma_{AB}$).

$$\delta_\xi C_{zz} = \left[\left(\mathcal{T} + \frac{u}{2}(\partial\mathcal{Y} + \bar{\partial}\bar{\mathcal{Y}}) \right) \partial_u + \mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial} + \frac{3}{2}\partial\mathcal{Y} - \frac{1}{2}\bar{\partial}\bar{\mathcal{Y}} \right] C_{zz} - 2\partial^2\mathcal{T} - u\,\partial^3\mathcal{Y} .$$

- With these identifications, the sourced Ward identities reproduce the BMS flux-balance laws.

Antipodal gluing

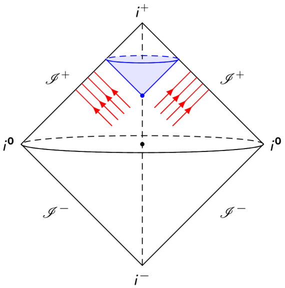
- Where does the Carrollian CFT live?
- Glue $\mathscr{I}^+$ and $\mathscr{I}^-$ by identifying antipodally $\mathscr{I}^+_-$ with $\mathscr{I}^-_+$:

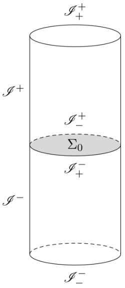
$$\boxed{\hat{\mathscr{I}} = \mathscr{I}^- \sqcup \mathscr{I}^+}$$

[Donnay-Herfray-Fiorucci-Ruzziconi '22]

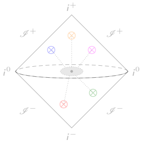
- Intrinsically, the gluing surface Σ_0 is distinguished by a vanishing n^a.
- Carrollian momenta and symmetry generators smoothly defined on $\hat{\mathscr{I}}$.

$\Longrightarrow$ Geometric implementation of the antipodal matching.
[Strominger '13] [Troessaert '17] [Henneaux-Troessaert '18] [Prabhu '19] [Capone-Nguyen-Parisini '22]

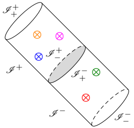




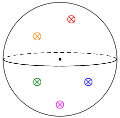
From the bulk...



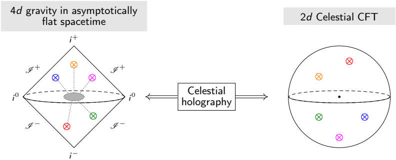
...to null infinity...



...to the celestial sphere.



Celestial holography



- Scattering of massless particles:
  [de Boer-Solodukhin '03] [Pasterski-Shao-Strominger '17] [Pasterski-Shao '17]

$$\langle \mathcal{O}_{\Delta_1,J_1}(z_1,\bar z_1)\dots\mathcal{O}_{\Delta_N,J_N}(z_N,\bar z_N)\rangle = \left(\prod_{i=1}^N \int_0^{+\infty} d\omega_i\, \omega_i^{\Delta_i-1}\right)\langle \text{out}|\mathcal{S}|\text{in}\rangle$$

  where the CCFT operators $\mathcal{O}_{\Delta_i,J_i}(z_i,\bar z_i)$ are characterized by conformal dimension Δ_i and spin J_i.
- Soft theorems in the bulk $\Longleftrightarrow$ Ward identities in the CCFT.
- The CCFT correlation functions obey the Ward identities: [Strominger '13] [Kapec-Mitra-Raclariu-Strominger '17]

$$\left\langle P(z,\bar z)\prod_{i=1}^N \mathcal{O}_{\Delta_i,J_i}(z_i,\bar z_i)\right\rangle + \hbar\sum_{q=1}^N \frac{1}{z-z_q}\left\langle \dots\mathcal{O}_{\Delta_q+1,J_q}(z_q,\bar z_q)\dots\right\rangle = 0 \quad \text{(leading soft theorem)} \quad P(z,\bar z):\left(\frac{3}{2},\frac{1}{2}\right)$$

$$\left\langle T(z)\prod_{i=1}^N \mathcal{O}_{\Delta_i,J_i}(z_i,\bar z_i)\right\rangle + \hbar\sum_{q=1}^N \left[\frac{\partial_q}{z-z_q}+\frac{h_q}{(z-z_q)^2}\right]\left\langle \prod_{i=1}^N \mathcal{O}_{\Delta_i,J_i}(z_i,\bar z_i)\right\rangle = 0 \quad \text{(subleading soft theorem)} \quad T(z):(2,0)$$

  where $h_q = \frac{1}{2}(\Delta_q + J_q)$.
- How to relate the $2d$ CCFT correlation functions and Ward identities with those of the $3d$ Carrollian CFT?

Relation between Carrollian and celestial operators

- Relation between (quasi) conformal Carrollian primary operators and CCFT operators [Donnay-Herfray-Fiorucci-Ruzziconi '22]:

$$\left.\begin{aligned} \mathcal{O}^{out}_{\Delta_i,J_i}(z_i,\bar{z}_i) &= i^{\Delta_i}\Gamma[\Delta_i]\int_{-\infty}^{+\infty} du_i\, u_i^{-\Delta_i}\, \Phi^{out}_{(k_i,\bar{k}_i)}(u_i,z_i,\bar{z}_i)\,, \\ \mathcal{O}^{in}_{\Delta_j,J_j}(z_j,\bar{z}_j) &= i^{\Delta_j}\Gamma[\Delta_j]\int_{-\infty}^{+\infty} dv_j\, v_j^{-\Delta_j}\, \Phi^{in}_{(k_j,\bar{k}_j)}(v_j,z_j,\bar{z}_j)\,. \end{aligned}\right\} \quad \text{(Fourier + Mellin transforms)}$$

$\Longrightarrow$ Exchange between time and conformal dimension.
$\Longrightarrow$ Extrapolate dictionary [Pasterski-Puhm-Trevisani '21].

- Matching between Carrollian weights $(k,\bar{k})$ and celestial spin J:

$$k = \frac{1}{2}(1+J)\,, \qquad \bar{k} = \frac{1}{2}(1-J)\,.$$

(compatible with the falloffs of the conformal compactification)

- Correlation functions ($N = m+n$):

$$\Big\langle \prod_{i=1}^{m} \mathcal{O}^{out}_{\Delta_i,J_i}(z_i,\bar{z}_i) \prod_{j=1}^{n} \mathcal{O}^{in}_{\Delta_j,J_j}(z_j,\bar{z}_j)\Big\rangle$$

$$= \left(\prod_{i=1}^{m} i^{\Delta_i}\Gamma[\Delta_i]\int_{-\infty}^{+\infty} du_i\, u_i^{-\Delta_i}\right)\left(\prod_{j=1}^{n} i^{\Delta_j}\Gamma[\Delta_j]\int_{-\infty}^{+\infty} dv_j\, v_j^{-\Delta_j}\right) \langle \underbrace{\Phi^{out}_{(k_1,\bar{k}_1)}(x_1)\dots\Phi^{out}_{(k_m,\bar{k}_m)}(x_m)}_{\text{Insertions at } \mathscr{I}^+}\underbrace{\Phi^{in}_{(k_1,\bar{k}_1)}(x_1)\dots\Phi^{in}_{(k_n,\bar{k}_n)}(x_n)}_{\text{Insertions at } \mathscr{I}^-}\rangle\,.$$

Relation between Carrollian and celestial Ward identities

- Integrated conformal Carrollian Ward identities:

$$\delta_{\bar{\xi}} \langle X \rangle = \frac{i}{\hbar} \left\langle \left(\int_{\mathscr{I}^- \sqcup \mathscr{I}^+} \mathbf{F}_{\bar{\xi}} - \int_{\mathscr{I}^+_+} \mathbf{j}_{\bar{\xi}} + \int_{\mathscr{I}^-_-} \mathbf{j}_{\bar{\xi}} \right) X \right\rangle$$

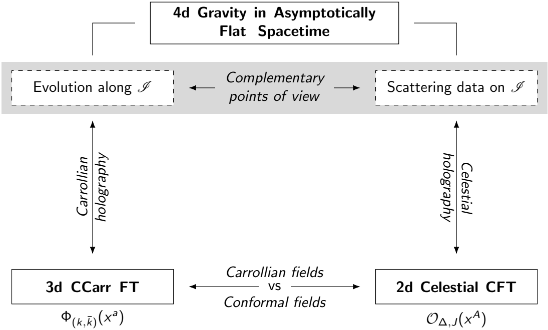
  where $X \equiv \Phi^{out}_{(k_1,\bar{k}_1)}(x_1) \dots \Phi^{out}_{(k_m,\bar{k}_m)}(x_m) \Phi^{in}_{(k_1,\bar{k}_1)}(x_1) \dots \Phi^{in}_{(k_n,\bar{k}_n)}(x_n)$.
- Assumption of massless scattering: $\mathbf{j}_{\bar{\xi}}|_{\mathscr{I}^+_+} = 0 = \mathbf{j}_{\bar{\xi}}|_{\mathscr{I}^-_-}$.
- Incoming flux = outgoing flux: $\int_{\mathscr{I}^-} \mathbf{F}_{\bar{\xi}} = - \int_{\mathscr{I}^+} \mathbf{F}_{\bar{\xi}}$ (constraint on the sources).
- With these assumptions $\Longrightarrow$ invariance of the correlators under BMS symmetries: $\boxed{\delta_{\bar{\xi}} \langle X \rangle = 0\,.}$

  [Donnay-Herfray-Fiorucci-Ruzziconi '22]

- Supertranslations:
  1. $\delta_{\mathcal{T}} \langle X \rangle = \delta^S_{\mathcal{T}} \langle X \rangle + \delta^H_{\mathcal{T}} \langle X \rangle$ ($\delta^S_{\mathcal{T}} \langle X \rangle \neq 0$ if there is at least one graviton insertion).
  2. $\delta^S_{\mathcal{T}} \langle X \rangle \sim \langle \int_{\mathscr{I}} F^S_{\mathcal{T}} X \rangle$ using $\left[\Pi_{zz}(u, z, \bar{z}), \Phi_{\bar{w}\bar{w}}(u', w, \bar{w}) \right] = 16\pi G \, i\hbar \, \delta(u - u') \, \delta^{(2)}(z - w)$ with $\Pi_{zz} = \partial_u \Phi_{zz}$.
  3. Specify the relation to $\mathcal{T}(z, \bar{z}) = \delta^{(2)}(z - w)$ and introduce the supertranslation current
  $$P(z, \bar{z}) = \frac{1}{4G} \left(\int_{-\infty}^{+\infty} du + \int_{-\infty}^{+\infty} dv \right) \bar{\partial} \Pi_{zz} .$$
  4. Perform the integral transforms on X:
  $$\boxed{\left\langle P(z, \bar{z}) \prod_{i=1}^{N} \mathcal{O}_{\Delta_i, J_i}(z_i, \bar{z}_i) \right\rangle + \hbar \sum_{q=1}^{N} \frac{1}{z - z_q} \left\langle \ldots \mathcal{O}_{\Delta_q + 1, J_q}(z_q, \bar{z}_q) \ldots \right\rangle = 0 .}$$
- Superrotations:
  1. $\delta_{\mathcal{Y}} \langle X \rangle = \delta^S_{\mathcal{Y}} \langle X \rangle + \delta^H_{\mathcal{Y}} \langle X \rangle$ ($\delta^S_{\mathcal{Y}} \langle X \rangle \neq 0$ if there is at least one graviton insertion).
  2. $\delta^S_{\mathcal{Y}} \langle X \rangle \sim \langle \int_{\mathscr{I}} F^S_{\mathcal{Y}} X \rangle$.
  3. Specify the relation to $\mathcal{Y}(z) = \frac{1}{z - w}$ and introduce the $2d$ stress-tensor
  $$T(z) = -\frac{i}{8\pi G} \int \frac{dw d\bar{w}}{z - w} \left(\int_{-\infty}^{+\infty} du \, u + \int_{-\infty}^{+\infty} dv \, v \right) \partial^3 \Pi_{\bar{w}\bar{w}} .$$
  4. Perform the integral transforms on X:
  $$\boxed{\left\langle T(z) \prod_{i=1}^{N} \mathcal{O}_{\Delta_i, J_i}(z_i, \bar{z}_i) \right\rangle + \hbar \sum_{q=1}^{N} \left[\frac{\partial_q}{z - z_q} + \frac{h_q}{(z - z_q)^2} \right] \left\langle \prod_{i=1}^{N} \mathcal{O}_{\Delta_i, J_i}(z_i, \bar{z}_i) \right\rangle = 0 .}$$

Summary

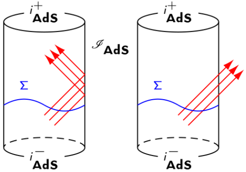
- Two complementary pictures of $\mathscr{I}^+$ and $\mathscr{I}^-$ leading to two complementary approaches of flat space holography:

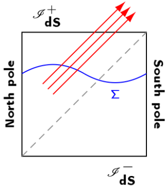


- Proposition of a dual theory in Carrollian holography: 3*d* Carrollian CFT coupled with external sources.
- The sources holographically encode the bulk radiation leaking through the boundary.
- Relation between Carrollian and celestial holographies established.
- Ward identities in CCarr FT $\Longleftrightarrow$ Ward identities in CCFT.

Perspectives

- From the bulk ... to null infinity ... to the celestial sphere.
  $\Longrightarrow$ Deduce more insights in Carrollian holography from celestial holography and reciprocally.
- Relation with (A)dS/CFT correspondence?
  $\Longrightarrow$ In the bulk, the flat limit works provided one starts with leaky boundary conditions.
  $\Longrightarrow$ Λ-BMS symmetries and phase space. [Compère-Fiorucci-Ruzziconi '19] [Fiorucci-Ruzziconi '21]
  $\Longrightarrow$ Obtain the sourced Carrollian CFT in the ultra-relativistic limit of a sourced conformal field theory.





- How general is our proposal?

Gravity in $d+1$ dimensions with leaky boundary conditions	$\Longleftrightarrow$	Sourced QFT in d dimensions

  $\Longrightarrow$ Holography of open systems.
  $\Longrightarrow$ Holography of finite regions.

Thank you!

Coadjoint representation of BMS_4

- In which representation does the solution space transform?
- In $3d$ asymptotically flat gravity, it transforms in the coadjoint representation of the BMS_3 algebra.

$\Longrightarrow$ Identify the coadjoint representation of the BMS_4 algebra in the transformation of the solution space of $4d$ gravity (momentum map)

- Short summary (see [Barnich-Ruzziconi '21] for details and more results):
  1. $(\mathcal{T}, \mathcal{Y}, \bar{\mathcal{Y}}) \in \mathfrak{bms}_4$ and $(p, [j], [\bar{j}]) \in \mathfrak{bms}_4^*$ with $j \sim j + \partial \mathcal{N}$ and $\bar{j} \sim \bar{j} + \bar{\partial} \bar{\mathcal{N}}$.
  2. Pairing: $\mathfrak{bms}_4^* \times \mathfrak{bms}_4 \mapsto \mathbb{R} : ((p, [j], [\bar{j}]), (\mathcal{T}, \mathcal{Y}, \bar{\mathcal{Y}})) \to \langle (\mathcal{P}, [\mathcal{J}], [\bar{\mathcal{J}}]), (\mathcal{T}, \mathcal{Y}, \bar{\mathcal{Y}}) \rangle$ with
$$\langle (p, [j], [\bar{j}]), (\mathcal{T}, \mathcal{Y}, \bar{\mathcal{Y}}) \rangle = \int_{\mathcal{S}} \frac{dz d\bar{z}}{(2i\pi)^2} [\mathcal{T} p + \mathcal{Y} \bar{j} + \bar{\mathcal{Y}} j] \, .$$
  3. The coadjoint representation ad^* is defined via $\langle ad^*_{(\mathcal{T}, \mathcal{Y}, \bar{\mathcal{Y}})}(p, [j], [\bar{j}]), (\mathcal{T}', \mathcal{Y}', \bar{\mathcal{Y}})] \rangle = -\langle (p, [j], [\bar{j}]), [(\mathcal{T}, \mathcal{Y}, \bar{\mathcal{Y}}), (\mathcal{T}', \mathcal{Y}', \bar{\mathcal{Y}}')] \rangle$, which implies
$$\delta_{(\mathcal{T}, \mathcal{Y}, \bar{\mathcal{Y}})} p = \mathcal{Y} \partial p + \bar{\mathcal{Y}} \bar{\partial} p + \frac{3}{2} \partial \mathcal{Y} p + \frac{3}{2} \bar{\partial} \bar{\mathcal{Y}} p,$$
$$\delta_{(\mathcal{T}, \mathcal{Y}, \bar{\mathcal{Y}})} j = \mathcal{Y} \partial j + \bar{\mathcal{Y}} \bar{\partial} j + \partial \mathcal{Y} j + 2 \bar{\partial} \bar{\mathcal{Y}} j + \frac{1}{2} \mathcal{T} \bar{\partial} p + \frac{3}{2} \bar{\partial} \mathcal{T} p.$$

$\Longrightarrow$ Construct the momentum map = identify $(p, [j], [\bar{j}])$ in the solution space of gravity.