Massive Higher Spins and Black Hole Interactions

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Plan of the talk

- Motivation
- Black Holes (reminder)
- Worldline description of Black Holes
- Spinor - Helicity Formalism
- Massive and Massless Higher Spin fields
- A cubic action for rotating Black Holes
- Summary and Conclusions

Based on

- E.D.Skvortsov, M.T., arXiv: 2312.08184

- A. Buonanno et al,
  Snowmass white paper: Gravitational Waves and Scattering Amplitudes, arXiv: 2204.05194
- P. Townsend,
  Black Holes, arXiv: gr-qc/9707012
- X. Bekaert, N. Boulanger, A. Campoleoni M.Chiodaroli, D. Francia, M. Grigoriev, E.Sezgin, E. Skvortsov,
  Snowmass White Paper: Higher Spin Gravity and Higher Spin Symmetry, arXiv:2205.01567
- A. Fotopoulos, M.T,
  Gauge Invariant Lagrangians for Free and Interacting Higher Spin Fields. A Review of the BRST Approach, arXiv: 0805.1346
- L. Dixon,
  Calculating Scattering Amplitudes Efficiently, arXiv: hep-ph/9601359
- S. Weinzierl
  Tales of 1001 Gluons, arXiv:1610.05318

Motivation

- Gravitational Waves (GW) are experimentally observed
- We need a theory which will be helpful for studies of GW
- Consider the interactions between to rotating (Kerr) Black Holes by exchange of Gravitational Waves
- It is complicated, if one uses the Einstein equations
- Use Effective Field Theory approach : model rotating Black Holes as localized massive objects (particles) with a large spin. It is possible, because of the "No - Hair" Theorem
- Use the Post Minkowskian Approximation i.e., weak fields $\frac{Gm}{rc^2} << \frac{v^2}{c^2}$ but $\frac{v^2}{c^2} \sim 1$. Perturb around special relativity (expansion in G)

- The most general solution of Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$$

with $T_{\mu\nu} = 0$ and spherical symmetry

$$ds^2 = -\left(1 - \frac{2MG}{r}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{2MG}{r}\right)} + r^2 (d\theta^2 + \sin^2\theta \, d\phi^2)$$

G - Newton constant, M - mass

- If the Schwarzschild radius $r_s = 2MG$ is bigger than the radius of the gravitating object, we have a Schwarzschild Black Hole. Event horizon is at r_s

- Radial propagation of the light

$$ds^2 = 0, \quad d\theta = d\phi = 0, \quad dt = \frac{dr}{\left(1 - \frac{2MG}{r}\right)}$$

Near horizon $t \sim 2MG \ln(r - 2MG) \to \infty$: It takes an infinite time for a light to reach the horizon

- If $dr = d\theta = d\phi = 0$, then

$$ds^2 = -\left(1 - \frac{2MG}{r}\right) dt^2 = -d\tau^2$$

τ is a proper time. The time measured by observer at infinity is bigger than the proper time: we have a time dilation.

- "No hair" theorem: Each Black Hole is described by its mass M, charge Q, coordinates x^i, momenta p_i and angular momenta $J^i{}_j$
- Rotating (Kerr) Black Hole

$$\begin{aligned} ds^2 &= -\frac{\Delta}{\rho^2}(dt - a\sin^2\theta\, d\phi)^2 + \frac{\sin^2\theta}{\rho^2}((r^2 + a^2)d\phi - adt)^2 + \\ &+ \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 \end{aligned}$$

where: $\Delta = r^2 - 2GMr + a^2, \quad \rho^2 = r^2 + a^2\cos^2\theta$

- Two parameters M- mass and $J = ma$ -angular momentum

- This approach has the following steps (J.Vines, arXiv: 1709.06016):
- Both for Schwarzschild and Kerr Black Holes. Let us consider the later.
- Linearize the metric around a flat background

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x) + \mathcal{O}(G^2)$$

- Here $g_{\mu\nu}(x)$ is the exact Kerr metric, $h_{\mu\nu}(x)$ is the exact solution of linearized field equations

$$\Box h_{\mu\nu} = -16\pi G \mathcal{P}_{\mu\nu\alpha\beta} T^{\alpha\beta}$$

and of the Lorentz gauge condition

$$\mathcal{P}_{\mu\nu}{}^{\alpha\beta} = \delta^{(\alpha}_{(\mu}\delta^{\beta)}_{\nu)} - \frac{1}{2}\eta_{\mu\nu}\eta_{\alpha\beta}$$

- The gauge invariance

$$\delta h_{\mu\nu} = \partial_\mu \lambda_\nu + \partial_\nu \lambda_\mu$$

is used to achieve the Lorentz gauge

- The action

$$S_{tot.} = S_{Grav.}[h] + S_{int.}[\Psi, h] + S_{kin.}[\Psi] + \mathcal{O}(G^2) =$$
$$= -\frac{1}{64\pi G}\int d^4x\,(\partial_\rho h_{\mu\nu})\mathcal{P}^{\mu\nu\alpha\beta}(\partial^\rho h_{\mu\nu}) + \frac{1}{2}\int d^4x\,h_{\mu\nu}T^{\mu\nu}[\Psi];$$
$$+S_{tot.}[\Psi, h=0] + \mathcal{O}(G^2)$$

- The variables Ψ describe an arbitrarily parametrized worldline $x^\mu = z^\mu(\tau)$, with tangent $\dot{z}^\mu = \frac{dz^\mu}{d\tau}$, along with some other variables $\psi(\tau)$
- Stress -energy tensor does not depend on $h_{\mu\nu}$

$$T^{\mu\nu}(x) = \int d\tau\,\tilde{T}^{\mu\nu}(\psi, \partial)\delta^4(x-z) + \mathcal{O}(G^2)$$

- $\tilde{T}^{\mu\nu}$ is a differential operator

$$\tilde{T}^{\mu\nu} = \dot{z}^{(\mu}p^{\nu)} + \dot{z}^{(\mu}S^{\nu)\alpha}\partial_\alpha + \ldots$$

  where p^μ and $S^{\mu\nu}$ are momentum and spin operators
- No other variables due to the "no hair" theorem

- Worldline fields obey a constraint

$$S^{\mu\nu}p_\nu = 0 \rightarrow S^{\mu\nu} = \varepsilon^{\mu\nu}{}_{\alpha\beta}a^\alpha p^\beta \quad \text{and} \quad a^\mu p_\mu = 0$$

  The vector a^μ is called a spin -vector

- Therefore, our ψ variables are p^μ and a^μ. Define the mass via $p^\mu = Mu^\mu$, where u^μ is a time-like unit vector
- The interaction term in the Lagrangian is

$$\mathcal{L}_{int.} = \frac{1}{2}\tilde{T}^{\mu\nu}(p, a, \partial)h_{\mu\nu}(x = z)$$

- The $h_{\mu\nu}$ for the Kerr metric is

$$h_{\mu\nu} = 4GM\mathcal{P}_{\mu\nu\alpha\beta}\exp{(a * \partial)^\alpha}_\gamma\frac{u^\gamma u^\beta}{r}, \quad (a * b)_{\mu\nu} = \varepsilon_{\mu\nu\alpha\beta}a^\alpha b^\beta$$

- From the equation

$$h_{\mu\nu} = 4G\mathcal{P}_{\mu\nu\alpha\beta}\,T^{\alpha\beta}(p, a, \partial)\frac{1}{r}$$

  We get: $\tilde{T}^{\mu\nu}(p, a, \partial) = M\exp{(a * \partial)^{(\mu}}_\rho u^{\nu)}u^\rho$

- We are again considering $D = 4$
- Light-like momentum is parametrized by commuting Weyl spinors λ_α and $\bar{\lambda}_{\dot{\alpha}}$, with $\alpha, \dot{\alpha} = 1, 2$ being $SL(2, C)$ group indices

$$p_{\alpha\dot{\alpha}} = \lambda_\alpha \bar{\lambda}_{\dot{\alpha}}, \quad p^{\alpha\dot{\alpha}} p_{\alpha\dot{\alpha}} = \varepsilon^{\alpha\beta} \varepsilon^{\dot{\alpha}\dot{\beta}} p_{\beta\dot{\beta}} p_{\alpha\dot{\alpha}} = 0$$

- Useful notations

$$|\lambda\rangle \leftrightarrow \lambda_\alpha, \quad |\lambda] \leftrightarrow \bar{\lambda}^{\dot{\alpha}}, \quad \langle\lambda| \leftrightarrow \lambda^\alpha, \quad [\lambda| \leftrightarrow \bar{\lambda}_{\dot{\alpha}}$$

$$\langle\lambda\rho\rangle = \lambda^\alpha \rho_\alpha, \quad [\lambda\rho] = \bar{\lambda}_{\dot{\alpha}} \bar{\rho}^{\dot{\alpha}}$$

- The vector fields are characterized by polarization vectors

$$\epsilon^+ = \sqrt{2} \frac{|q\rangle[\lambda|}{\langle q\lambda\rangle}, \quad \epsilon^- = \sqrt{2} \frac{|\lambda\rangle[q|}{[\lambda q]}$$

where $|q\rangle$ and $|q]$ are arbitrary reference spinors. Their presence is a result of the gauge invariance.

- The reference spinors $|q\rangle$ and $|q]$ can be chosen for each external particle separately.
- This formalism greatly simplifies calculations of scattering amplitudes
- Example : a scattering of n positive helicity massless vector fields.
- We have at most $n-2$ Yang-Mills vertices. Each vertex has at most one momentum. Therefore we have at least one contraction

$$\epsilon_i^+(q) \cdot \epsilon_j^+(q)$$

- Choosing the reference spinor equal to all external particles we get $\epsilon_i^+(q) \cdot \epsilon_j^+(q) = 0$, therefore

$$\mathcal{A}(1^+, 2^+, ..., n^+) = 0$$

- The first nonzero amplitude (so called MHV)

$$\mathcal{A}(1^-, .j^-, l^-.., n^+) = \frac{\langle jl \rangle^4}{\langle 12 \rangle \cdots \langle n1 \rangle}$$

- The four - momentum with $p^2 = m^2$ is parametrized by Dirac spinors λ_α^a and $\bar{\lambda}_{\dot{\alpha}}^a$, with $a = 1, 2$ being $SU(2)$ group index

$$p_{\alpha\dot{\alpha}} = \lambda_\alpha^a \bar{\lambda}_{\dot{\alpha},a}$$

$$|\lambda_a\rangle_\alpha \langle^a \lambda|^\beta = m\delta_\alpha^\beta, \quad \langle \lambda^a \lambda^b \rangle = -m\varepsilon^{ab}$$

- It is useful to introduce auxiliary commuting variables z_a and consider $\lambda_\alpha = \lambda_\alpha^a z_a$, and $\bar{\lambda}_{\dot{\alpha}} = \bar{\lambda}_{\dot{\alpha}}^a z_a$
- The massive spinors satisfy the Dirac equations

$$p_{\alpha\dot{\alpha}} \bar{\lambda}^{\dot{\alpha}} = m\lambda_\alpha, \quad p_{\alpha\dot{\alpha}} \lambda^\alpha = -m\bar{\lambda}_{\dot{\alpha}},$$

- For massive vector fields we have polarization vectors

$$\epsilon = \sqrt{2}\frac{|\lambda\rangle[\lambda|}{m}$$

- For an arbitrary spin s field the polarization tensor is

$$\epsilon^{(s)} = (\epsilon)^s$$

- Three point functions (N. Arkani-Hamed, T.-C. Huang, Y.-t. Huang, arXiv:1709.04891, (AHH))
- One massless field with a helicity h (label '3') and two massive fields with equal mass m and spin s (labels '1' and '2').
- Momentum conservation implies

$$2p^{(1)} \cdot p^{(3)} = \langle 3|p^{(1)}|3] = 0$$

- Therefore

$$x\lambda^{(3),\alpha} = \bar{\lambda}^{(3)}_{\dot{\alpha}} \frac{p^{(1),\alpha\dot{\alpha}}}{m}$$

- Introducing an auxiliary spinor q^α one can write

$$mx\langle 3q\rangle = \langle q|p_1|3]$$

- Identifying q^α with the reference spinor for massless fields we get

$$\frac{\sqrt{2}}{m}\epsilon^{(3),+} \cdot p^{(1)} = x, \quad \frac{\sqrt{2}}{m}\epsilon^{(3),-} \cdot p^{(1)} = \frac{1}{x}$$

- A three point function can be expanded in the basis of $\lambda^{(3)}_\alpha$ and $\varepsilon_{\alpha\beta}$
- The general expansion has the form

$$M_3^{(\alpha_1,\ldots,\alpha_s),(\beta_1,\ldots,\beta_s),h} =$$
$$= (mx)^h \left(\sum_{t=0}^{2s} g_t \varepsilon^{2s-a} \left(x \frac{\lambda^{(3)}\lambda^{(3)}}{m} \right)^t \right)^{(\alpha_1,\ldots,\alpha_s),(\beta_1,\ldots,\beta_s)}$$

- Or reintroducing the massive spinors

$$M_3^{s,s,h} = (mx)^h \left(g_0 \frac{\langle 21\rangle^{2s}}{m^{2s}} + g_1 x \frac{\langle 21\rangle^{2s-1}\langle 23\rangle\langle 31\rangle}{m^{2s+1}} + \ldots \right)$$

- The requirement that the amplitude has a good Ultraviolet behaviour, puts all $g_t = 0$, except g_0. This is called the minimal coupling

- We shall consider $h = 2$ (gravity) and $h = 1$ (vector field)

$$M_3^{s,s,2} = (mx)^2 \frac{\langle 21 \rangle^{2s}}{m^{2s}}, \quad M_3^{s,s,1} = (mx) \frac{\langle 21 \rangle^{2s}}{m^{2s}},$$

- For the Kerr black hole, after the Fourier transformation

$$T^{\mu\nu}(-k) = 2\pi\delta(p \cdot k) p^{(\mu} \exp\left(\frac{S * ik}{m}\right)^{\nu)}_{\rho} p^{\rho}$$

with $(S * ik)^{\mu}{}_{\nu} = \varepsilon^{\mu}{}_{\nu\rho\sigma} S^{\rho} k^{\sigma}$

- The cubic interaction with graviton is

$$V_{3.gr} = \varepsilon^{\mu}(k)\varepsilon^{\nu}(k) T^{\mu\nu}(-k)$$

- Taking carefully the classical limit of $M_3^{s,s,2}$, one obtains $V_{3.gr}$.

- A possible way (M.Chiodaroli, H. Johansson, P. Pichini, arXiv: 2107.14779):
- Rewrite the AHH amplitude in an exponent-like form

$$M_3^{s,s,2} = -i(\varepsilon \cdot p_1)^2 \left(\varepsilon_2 \cdot \left(1 + \frac{k \cdot \hat{S}}{m} + \frac{(k \cdot \hat{S})^2}{m^2} \right) \cdot \varepsilon_1 \right)^s$$

where $\hat{S}^\mu$ is a spin operator

$$(\hat{S}^\mu)_{\alpha\beta} = \epsilon^{\mu\nu\rho\tau} p_{1,\nu} (M_{\rho\tau})_{\alpha\beta}, \quad (M^{\mu\nu})_{\alpha\beta} = 2i\delta_\alpha^{[\mu}\delta_\beta^{\nu]}$$

- Define the classical spin vector as an expectation value

$$\varepsilon_2 \cdot \hat{S}^\mu \cdot \varepsilon_1 \sim \frac{S^\mu}{s}$$

- Finally take limits $k << p$ and $s \to \infty$ and get $V_{3.gr}$

- What is an off-shell description of the results above?
- Higher Spin fields are usually described by symmetric tensors of rank s i.e., in terms of $\phi_{\mu_1\mu_2,\ldots,\mu_s}(x)$
- The tensors can be traceless (irreducible) or traceful (reducible) representations of the Poincarè (or AdS_D, dS_D) group
- We use a gauge invariant approach, similar to the Open String Field Theory
- Free equations

$$
\begin{aligned}
&(\Box - m^2)\phi_{\mu_1\mu_2,\ldots,\mu_s}(x) = 0, && \text{mass-shell}\\
&\partial^\mu \phi_{\mu\mu_2,\ldots,\mu_s}(x) = 0, && \text{transverse}\\
&\phi^\mu{}_{\mu\mu_3,\ldots,\mu_s}(x) = 0 && \text{traceless}
\end{aligned}
$$

- We would like to construct an action which gives these conditions as equations of motion
- We shall not impose the zero trace condition

- Introduce an auxiliary Fock space spanned by oscillators

$$[\alpha_\mu, \alpha_\nu^+] = \eta_{\mu\nu}, \quad [\alpha_D, \alpha_D^+] = 1$$

- A vector in the Fock space

$$|\Phi\rangle = \sum_{k=0}^{k=s} \frac{1}{(s-k)!k!} \Phi_{\mu_1\mu_2,\dots\mu_{s-k}}(x)\alpha^{\mu_1,+}\alpha^{\mu_2,+}\dots\alpha^{\mu_s,+}(\alpha_D^+)^k|0\rangle,$$

- Mass-shell and transversality conditions

$$l_0|\varphi\rangle = 0, \quad l|\varphi\rangle = 0$$

with

$$l_0 = p\cdot p + m^2, \quad l = p\cdot\alpha + m\alpha_D, \quad l^+ = p\cdot\alpha^+ + m\alpha_D^+, \qquad p_\mu = -i\partial_\mu$$

- Introduce ghost variables

$$\{c_0, b_0\} = \{c, b^+\} = \{c^+, b\} = 1$$

- The field has the form: (s is a total number of α_μ^+ and α_D^+ oscillators)

$$|\Phi\rangle = |\varphi^{(s)}\rangle + c^+b^+|D^{(s-2)}\rangle + c_0b^+|C^{(s-1)}\rangle$$

- The physical field is $|\varphi^{(s)}\rangle$, the fields $|D^{(s-2)}\rangle$ and $|C^{(s-1)}\rangle$ are gauge artefacts
- The only nonzero commutator between the operators is

$$[l, l^+] = l_0$$

- The corresponding nilpotent BRST charge $Q^2 = 0$

$$Q = c_0 l_0 + c^+ l + c l^+ - c^+ c\, b_0,$$

- Since $Q^2 = 0$, a free action

$$\mathcal{L}_2 = \int dc_0 \langle\Phi|Q|\Phi\rangle, \quad \int dc_0\, c = 1$$

is invariant under gauge transformations

$$\delta|\Phi\rangle = Q|\Lambda\rangle, \quad |\Lambda\rangle = b^+|\lambda\rangle$$

- An example: massive spin 1. The field $\phi_\mu(x)$ is physical, $\phi(x)$ and $C(x)$ are auxiliary
- The fields and the parameter of gauge transformations

$$|\Phi\rangle = \left(\phi_\mu(x)\alpha^{\mu+} + i\phi(x)\alpha_D^+ - ic_0 b^+ C(x)\right)|0\rangle, \quad |\Lambda\rangle = ib^+\lambda(x)|0\rangle$$

- The corresponding Lagrangian

$$\mathcal{L} = \phi^\mu(\Box - m^2)\phi_\mu + \phi(\Box - m^2)\phi - C^2 + 2C\partial^\mu\phi_\mu - 2mC\phi$$

- Gauge transformations

$$\delta\phi_\mu(x) = \partial_\mu\lambda(x), \quad \delta\phi(x) = m\lambda(x), \quad \delta C(x) = (\Box - m^2)\lambda(x)$$

- After fixing the gauge one can see, that the system describes $D-1$ on-shell degrees of freedom

Massless Higher Spin Fields

- The construction is similar to the massive case. To describe free massless fields, put $m^2 = 0$ and discard α_D^+ dependence everywhere
- Fock space state

$$|\Phi\rangle = \frac{1}{s!}\Phi_{\mu_1\mu_2,\ldots\mu_s}(x)\alpha^{\mu_1,+}\alpha^{\mu_2,+}\ldots\alpha^{\mu_s,+}|0\rangle, \quad [\alpha_\mu, \alpha_\nu^+] = \eta_{\mu\nu}$$

satisfies mass-shell and transversality conditions

$$l_0|\varphi\rangle = 0, \quad l|\varphi\rangle = 0$$

with

$$l_0 = p\cdot p, \quad l = p\cdot\alpha, \quad l^+ = p\cdot\alpha^+, \quad p_\mu = -i\partial_\mu$$

- The field has the form: (s is a number of α_μ^+ oscillators)

$$|\Phi\rangle = |\varphi^{(s)}\rangle + c^+b^+|D^{(s-2)}\rangle + c_0b^+|C^{(s-1)}\rangle$$

- The nilpotent BRST charge

$$Q = c_0 l_0 + c^+ l + c l^+ - c^+ c b_0,$$

gives a free Lagrangian $\mathcal{L} \sim \langle\Phi|Q|\Phi\rangle$

- OUR TASK: describe cubic interactions between two Higher Spin fields with equal mass m, with massless spin two or spin one
- Restrict $s = 2$ or $s = 1$ for massless fields
- Take three copies of oscillators

$$[\alpha_\mu^{(i)}, \alpha_\nu^{(j),+}] = \eta_{\mu\nu}\delta^{ij}$$

  take $i = 1, 2$ for massive, $i = 3$ for massless fields.
- Consider nonlinear gauge transformations

$$\delta_{cub.}|\Phi^{(1)}\rangle \sim Q^{(1)}|\Lambda^{(1)}\rangle - g(\langle\Phi^{(2)}|\langle\Lambda^{(3)}| + \langle\Lambda^{(2)}|\langle\Phi^{(3)}|)|V_3\rangle)$$

- The corresponding cubic Lagrangian is

$$\mathcal{L}_{cub.} \sim \sum_{i=1}^{3}\langle\Phi^{(i)}|Q^{(i)}|\Phi^{(i)}\rangle + g\langle\Phi^{(3)}|\langle\Phi^{(2)}|\langle\Phi^{(1)}||V_3\rangle$$

- The invariance of $\mathcal{L}_{cub.}$:

$$g^0 : \qquad (Q^{(1)})^2 = (Q^{(2)})^2 = (Q^{(3)})^2 = 0$$
$$g^1 : \qquad (Q^{(1)} + Q^{(2)} + Q^{(3)})|V_3\rangle = 0$$

- Complete classification of the BRST invariant cubic vertices is known (R. Metsaev, arXiv: 1205.3131)
- They are given in terms of the BRST invariant expressions

$$\mathcal{K}^{(1)} = (p^{(2)} - p^{(3)}) \cdot \alpha^{(1),+} + m\alpha_D^{(1),+} + gh.$$

$$\mathcal{K}^{(2)} = (p^{(3)} - p^{(1)}) \cdot \alpha^{(2),+} - m\alpha_D^{(2),+} + gh.$$

$$\mathcal{K}^{(3)} = (p^{(1)} - p^{(2)}) \cdot \alpha^{(3),+} + gh.$$

$$\mathcal{Q} = \alpha^{(1),+} \cdot \alpha^{(2),+} + \alpha_D^{(1),+}\alpha_D^{(2),+} + \frac{\alpha_D^{(1),+}\mathcal{K}^{(2)} - \alpha_D^{(1),+}\mathcal{K}^{(2)}}{2m} + gh.$$

$$\mathcal{Z} = (\alpha^{(1),+} \cdot \alpha^{(2),+} + \alpha_D^{(1),+}\alpha_D^{(2),+})\mathcal{K}^{(3)} + (\alpha^{(2),+} \cdot \alpha^{(3),+})\mathcal{K}^{(1)} + \\ +(\alpha^{(3),+} \cdot \alpha^{(1),+})\mathcal{K}^{(2)} + gh.$$

- Apparently, any function of $\mathcal{K}^{(i)}$, $\mathcal{Q}$ and $\mathcal{Z}$ is a valid cubic vertex

- Our task is to find a function of $\mathcal{K}^{(i)}$, $\mathcal{Q}$ and $\mathcal{Z}$, which gives the minimal coupling
- Let us consider the physical ($\alpha_\mu^{(i),+}$ only) part of the cubic vertices
- The simplest example is $0-0-1$ case. The fields are

$$\langle\phi^{(1)}| = \langle 0|\phi^{(1)}, \quad \langle\phi^{(2)}| = \langle 0|\phi^{(2)}, \quad \langle\phi^{(3)}| = \langle 0|\alpha_\mu^{(3)}\epsilon_\mu^-$$

- The relevant cubic vertex has the form

$$V = \mathcal{K}^{(3)}$$

- The three point amplitude is

$$A_{\phi\phi A} = \frac{1}{2}\epsilon^- \cdot (p^{(1)} - p^{(2)}) = \epsilon^- \cdot p^{(1)} = \frac{-imx}{2}$$

- Similarly, one can find that A_{1-1-1}, A_{1-1-2} and A_{2-2-2} are generated by $\mathcal{Z}$, $\mathcal{Z}\mathcal{K}^{(3)}$ and $\mathcal{Z}^2$ respectively.

- The example $3-3-2$ tells us, that the corresponding vertex is

$$V = -\frac{1}{2}\left(\mathcal{Z}^2\mathcal{Q} - \mathcal{Z}\mathcal{Q}^2\mathcal{K}^{(3)} + \mathcal{Z}^2\mathcal{K}^{(1)}\mathcal{K}^{(2)}\frac{1}{2m^2}\right)$$

- Only some specific coefficients between the BRST invariant expressions give the minimal coupling
- Generic solution (generalizes an on-shell result of M.Chiodaroli, H.Johansson and P.Pichini, arXiv: 2107.14779)
- For $s-s-2$

$$V(s,s,2) = -\frac{i}{2}(\mathcal{K}^{(3)})^2 - \frac{1}{2}\left(\mathcal{Z}\mathcal{K}^{(3)} + \frac{\mathcal{Z}^2 - \mathcal{Q}^2\mathcal{K}^{(3)}\mathcal{Z}}{(1-\mathcal{Q})^2 - \frac{1}{2m^2}\mathcal{K}^{(1)}\mathcal{K}^{(2)}}\right)$$

- For $s-s-1$ (so called root Kerr, useful for the double copy construction)

$$V(s,s,1) = -\frac{i}{\sqrt{2}}\mathcal{K}^{(3)} - \frac{i}{\sqrt{2}}\frac{\mathcal{Z} - \mathcal{Q}^2\mathcal{K}^{(3)}}{(1-\mathcal{Q})^2 - \frac{1}{2m^2}\mathcal{K}^{(1)}\mathcal{K}^{(2)}}$$

- We considered an Effective Field Theory Approach for a graviton (Electromagnetic field) interacting with a Rotating Black Hole
- To this end we used a gauge invariant approach to Massive and Massless Higher Spin fields
- In particular, we used the BRST formalism, which is completely off-shell
- It produces a Lagrangian completion to the on-shell AHH vertex
- The approach is simple, since the vertices and BRST charges for reducible fields are used

THANK YOU!!!