Tuple Relational Calculus

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Motivation

S[S#, SNAME, STATUS, CITY]
P[P#, PNAME, COLOR, WEIGHT, CITY]
SP[S#, P#, QTY)]

Get all pairs of city names such that a supplier located in the first city supplies a part stored in the second city.

```
SELECT   s.CITY, p.CITY
FROM     S AS s, SP AS r, P AS p
WHERE    s.S# = r.S#
AND      r.P# = p.P# ;
```

$$\{s.4, p.5 \mid S(s) \wedge P(p) \wedge \exists r(SP(r) \wedge s.1 = r.1 \wedge r.2 = p.1)\}$$

$$\{s.4, p.5 \mid \exists r(S(s) \wedge P(p) \wedge SP(r) \wedge s.1 = r.1 \wedge r.2 = p.1)\}$$

Motivation

Get supplier names for suppliers who supply all red parts.

```
SELECT   s.SNAME
FROM     S AS s
WHERE    NOT EXISTS (   SELECT   *
                        FROM     P AS p
                        WHERE    p.COLOR = 'Red'
                        AND      NOT EXISTS (   SELECT   *
                                                FROM     SP AS r
                                                WHERE    r.S# = s.S#
                                                AND      r.P# = p.P# ) ) ;
```

$$\{s.2 \mid S(s) \wedge \forall p \in P\big(p.3 = \text{`red'} \rightarrow \exists r \in SP(r.1 = s.1 \wedge r.2 = p.1)\big)\}$$

$$\{s.2 \mid S(s) \wedge \neg\exists p \in P\big(p.3 = \text{`red'} \wedge \neg\exists r \in SP(r.1 = s.1 \wedge r.2 = p.1)\big)\}$$

Formulas

Alphabet

- Relation names $R, S, T, \ldots$, each of fixed arity in $\{1, 2, \ldots\}$.
- Tuple variables $r, s, t, \ldots$, each of fixed arity.

Terms

- Every constant is a term.
- If r is a tuple variable of arity n and $i \in \{1, 2, \ldots, n\}$, then $r.i$ is a term.

Atomic formulas

- If R is a relation name and r a tuple variable, both of the same arity, then $R(r)$ is an atomic formula.
- If u_1 and u_2 are terms, then $u_1 = u_2$ is an atomic formula.

Formulas

- Every atomic formula is a formula.
- If φ_1 and φ_2 are formulas, then $\neg\varphi_1$, $\varphi_1 \wedge \varphi_2$, $\varphi_1 \vee \varphi_2$ are formulas.
- If φ is a formula with free tuple variable r, then $\exists r\varphi$ and $\forall r\varphi$ are formulas.

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Queries

A query is an expression of the form

$$\{L \mid \varphi\}$$

where

- L is a list of terms;
- φ is a formula;
- whenever $r.i$ is a term in L, then r is a free tuple variable of φ.

Abbreviations

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- $\varphi_1 \to \varphi_2$ is an abbreviation for $\neg\varphi_1 \vee \varphi_2$
- $\exists r \in R(\varphi)$ is an abbreviation for $\exists r(R(r) \wedge \varphi)$
- $\forall r \in R(\varphi)$ is an abbreviation for $\forall r(R(r) \to \varphi)$
- $u_1 \neq u_2$ is an abbreviation for $\neg(u_1 = u_2)$

Notice that these abbreviations make sense:

$$
\begin{aligned}
\forall r \in R(\varphi) &\equiv \neg\neg\forall r \in R(\varphi) \\
&\equiv \neg\neg\forall r(R(r) \to \varphi) \\
&\equiv \neg\exists r\neg(\neg R(r) \vee \varphi) \\
&\equiv \neg\exists r(R(r) \wedge \neg\varphi) \\
&\equiv \neg\exists r \in R(\neg\varphi)
\end{aligned}
$$

Semantics

- A tuple variable of arity n ranges over $\mathbf{dom}^n$.
- $R(r)$ is true if tuple r belongs to relation R.
- $\exists r\varphi$ is true if there exists $r \in \mathbf{dom}^n$ that makes φ true (where n is the arity of r).
- ...
- In tuple relational calculus, we also have the problem of domain dependence.

$$\{r.1 \mid R(r) \vee \exists s(S(s))\}$$

$$\{r.1 \mid \neg R(r)\}$$

- How to express $\{r.1 \mid R(r) \vee S(r)\}$ in SQL?
  SQL is a mix of tuple relational calculus and relational algebra.

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Exercise

Get pairs (n_1, n_2) of supplier names such that the parts supplied by n_1 is a subset of the parts supplied by n_2.

$$\{s_1.2, s_2.2 \mid S(s_1) \wedge S(s_2) \wedge \forall r_1 \in SP \\ (r_1.1 = s_1.1 \rightarrow \exists r_2 \in SP(r_2.1 = s_2.1 \wedge r_2.2 = r_1.2))\}$$

$$\{s_1.2, s_2.2 \mid S(s_1) \wedge S(s_2) \wedge \neg\exists r_1 \in SP \\ (r_1.1 = s_1.1 \wedge \neg\exists r_2 \in SP(r_2.1 = s_2.1 \wedge r_2.2 = r_1.2))\}$$

```
SELECT   s1.SNAME, s2.SNAME
FROM     S AS s1, S AS s2
WHERE    NOT EXISTS (   SELECT   *
                        FROM     SP AS r1
                        WHERE    r1.S# = s1.S#
                        AND      NOT EXISTS (   SELECT   *
                                                FROM     SP AS r2
                                                WHERE    r2.S# = s2.S#
                                                AND      r2.P# = r1.P# ) ) ;
```