

RÉSUMÉS DES EXPOSÉS

SLAAG - 13 JANVIER 2026

Lorenzo Furio : *Recent progress on adelic Galois images for elliptic curves over $\mathbb{Q}$ - part I*

Let E be an elliptic curve defined over $\mathbb{Q}$. When E does not have potential complex multiplication, Serre’s open image theorem asserts that the Galois action on the torsion points of E is “as large as possible” : the image of the adelic Galois representation is open in $\mathrm{GL}_2(\widehat{\mathbb{Z}})$. In 1978, Mazur proposed a far-reaching refinement, his “Program B”, calling for a complete classification of all possible adelic images of Galois for elliptic curves over $\mathbb{Q}$.

Over the last 15 years, there has been striking progress towards this goal, ultimately depending on our ability to find rational points on certain algebraic curves over $\mathbb{Q}$ known as *modular curves*. In this first talk, I will introduce the problem, survey the state of the art, explain how it splits naturally into subcases, and describe the remaining open cases. These involve in particular a class of subgroups known as normalisers of non-split Cartans.

I will also start discussing recent results, joint with Matthew Bisatt and Davide Lombardo, which bring two new techniques to bear on the problem : p -adic Hodge theory and the reduction to generalised Fermat equations. Finally, I will give some details about the latter in the case of the normaliser of the non-split Cartan modulo 49.

Davide Lombardo : *Recent progress on adelic Galois images for elliptic curves over $\mathbb{Q}$ - part II*

In this second part, we expand on the results introduced in the first half of the talk, focussing on the local aspect of Mazur’s program B and studying the possible p -adic images of Galois for elliptic curves over $\mathbb{Q}$.

After giving more details on our analysis of generalised Fermat equations, I will describe ongoing joint work with Matthew Bisatt and Lorenzo Furio, showing that for all primes $p > 37$, the only possible p -adic images of Galois are the inverse images in $\mathrm{GL}_2(\mathbb{Z}_p)$ of a non-split Cartan subgroup modulo p^n for some $n \geq 1$.

The proof requires excluding both proper subgroups of the non-split Cartan modulo p and certain “exotic” groups arising at level p^2 . To rule out the latter, we use tools from p -adic Hodge theory to obtain an explicit description of the p^2 -torsion representation of elliptic curves over $\mathbb{Q}_p$ in the most delicate case – bad, potentially good supersingular reduction.