Groupe de Contact FNRS - Analyse Fonctionnelle
FNRS Group - Functional Analysis
Mons, June 12-13, 2023

Program

Monday, June 12, 2023

12:00	–	reception of participants (with sandwiches)
13:15-13:30	–	opening
13:30-14:20	–	**Jürgen Müller** Dynamics of the Backward Taylor Shift on Bergman Spaces
14:20-15:10	–	**Antoni López Martínez** The $T \oplus T$-recurrence problem
15:10-15:40	–	coffee break
15:40-16:30	–	**Dorothee Haroske** Morrey smoothness spaces
16:30-17:20	–	**Krzysztof Piszczek** Amenability of Köthe sequence algebras of order zero and infinity
19:30	–	restaurant L'Excelsior (Grand Place Mons)

Tuesday, June 13, 2023

9:00- 9:50	–	**Chiara Boiti** Mean-dispersion principles and the Wigner transform
9:50-10:40	–	**Armin Rainer** On polynomial-like properties of differentiable functions
10:40-11:10	–	coffee break
11:10-12:00	–	**Fernando Costa Jr.** Self-similar fractals and common hypercyclicity
12:00-12:50	–	**Gilles Godefroy** Some remarks on subspaces of L_∞
13:00	–	closing (with sandwiches)

Abstracts

Mean-dispersion principles and the Wigner transform

June 13
9:00-9:50

Chiara Boiti
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Joint work with: David Jornet (Universitat Politècnica de Valencia, Spain) and Alessandro Oliaro (Università di Torino, Italy)

We prove an uncertainty principle for an orthonormal sequence $\{f_k\}_{k\in\mathbb{N}_0}$ in $L^2(\mathbb{R})$ related to the Wigner transform

$$W(f_k)(x,\xi) = \frac{1}{\sqrt{2\pi}} \int f_k\left(x+\frac{t}{2}\right) \overline{f_k\left(x-\frac{t}{2}\right)} e^{-it\xi} dt.$$

More precisely, given the following operator (dual, by means of Fourier transform, to the twisted laplacian operator)

$$\hat{L} := \left(\frac{1}{2}D_\xi + x\right)^2 + \left(\frac{1}{2}D_x - \xi\right)^2,$$

we have that

$$\sum_{k=0}^{n} \langle \hat{L}W(f_k), W(f_k)\rangle \geq (n+1)^2, \qquad \forall n \in \mathbb{N}_0.$$

Moreover equality holds for $0 \leq n \leq n_0$, if and only if all f_k with $0 \leq k \leq n_0$ are equal, up to a unitary constant, to the Hermite functions h_k defined by

$$h_k(t) = \frac{1}{(2^k k! \sqrt{\pi})^{1/2}} e^{-t^2/2} H_k(t), \quad H_k(t) = (-1)^k e^{t^2} \frac{d^k}{dt^k} e^{-t^2}, \quad k \in \mathbb{N}_0.$$

The above uncertainty principle implies, as a trivial consequence, *Shapiro's mean-dispersion principle*: there does not exist an infinite orthonormal sequence $\{f_k\}_{k\in\mathbb{N}_0}$ in $L^2(\mathbb{R})$ such that all means $\mu(f_k)$, $\mu(\hat{f}_k)$ and variances $\Delta(f_k)$, $\Delta(\hat{f}_k)$ associated to f_k and their Fourier transform $\hat{f}_k$ are uniformly bounded. Here

$$\mu(f) := \frac{1}{\|f\|^2} \int t|f(t)|^2 dt, \quad \Delta(f) := \frac{1}{\|f\|} \left(\int |t-\mu(f)|^2 |f(t)|^2 dt \right)^{1/2}, \quad f \in L^2(\mathbb{R}).$$

June 13
11:10-12:00

Self-similar fractals and common hypercyclicity

Fernando Costa Jr.

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In 2003, E. Abakumov and J. Gordon proved that there exists a vector $x \in \ell_1$ which is simultaneously hypercyclic for every Rolewicz operator $\lambda B : \ell_1 \to \ell_1$, $\lambda > 1$ (here, B is the canonical backward shift). This result constitute the start point of the common hypercyclicity problem. Right after (in 2004), G. Costakis and M. Sambarino obtained a very general and practical criterion for common hypercyclicity, with applications to the family $(\lambda B)_{\lambda>1}$ and many others. In the core of these results is a special discretization of the parameter set indexing the family (in the form of a covering with particular properties). Since then, the problem concerning sets of parameter of higher dimensions have been studied by many authors and a generalization of the Costakis-Sambarino criterion, with applications to sets with fractal dimension, was obtained in 2022 by Q. Menet, F. Bayart and myself. This generalization, although very general, is not optimal. In this talk, we present a new way of discretizing a self-similar set of parameters, which allows us to promote the previous generalization to optimal. Applications include many self-similar fractals and any Hölder curve.

June 13
12:00-12:50

Some remarks on subspaces of L_∞

Gilles Godefroy

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The antidifferentiation operator is shown to provide examples of norm-closed subspaces of L_∞ which are Borel subsets of arbitrarily high rank for the weak-star topology. However, it is not known if a weak-star Borel subspace of L_∞ which is isomorphic to L_∞ is necessarily weak-star closed, and the similar problem for l_∞ is also open. We will provide some partial positive answers to these problems.

Morrey smoothness spaces

June 12
15:40-16:30

Dorothee Haroske
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In the recent years so-called Morrey smoothness spaces attracted a lot of interest. They can (also) be understood as generalisations of the classical spaces Besov and Triebel-Lizorkin spaces $A^s_{p,q}(\mathbb{R}^n)$, $A \in \{B, F\}$, where the parameters satisfy $s \in \mathbb{R}$ (smoothness), $0 < p \leq \infty$ (integrability) and $0 < q \leq \infty$ (summability). In the case of Morrey smoothness spaces additional parameters are involved. In our opinion, among the various approaches at least two scales enjoy special attention, also in view of applications: the scales $\mathcal{A}^s_{u,p,q}(\mathbb{R}^n)$, with $\mathcal{A} \in \{\mathcal{N}, \mathcal{E}\}$, $u \geq p$, and $A^{s,\tau}_{p,q}(\mathbb{R}^n)$, with $\tau \geq 0$.

I will present some important features and interesting results obtained in the recent past, with some references to the motivation and background material. Quite recently we also managed to reorganise these two prominent types of Morrey smoothness spaces by adding to (s, p, q) the so-called slope parameter ϱ, preferably (but not exclusively) with $-n \leq \varrho < 0$. It comes out that $|\varrho|$ replaces n, and $\min(|\varrho|, 1)$ replaces 1 in slopes of (broken) lines in the $(\frac{1}{p}, s)$-diagram characterising distinguished properties of the spaces $A^s_{p,q}(\mathbb{R}^n)$ and their Morrey counterparts. We think that (to some extent) this new approach helps to understand the influence of the Morrey parameters in the definition of the spaces.

The talk is based on joint work with Leszek Skrzypczak (Poznan), Susana Moura (Coimbra), and Hans Triebel (Jena).

The $T \oplus T$-recurrence problem

June 12
14:20-15:10

Antoni López Martínez
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Let X be a separable infinite-dimensional F-space. Given a continuous linear operator $T : X \longrightarrow X$ with some dynamical *"property"* (hypercyclicity, chaos, etc.), it is natural to ask whether the direct sum operator

$$T \oplus T : X \oplus X \longrightarrow X \oplus X,$$

acting as $T \oplus T(x_1, x_2) := (Tx_1, Tx_2)$ on the direct sum space $X \oplus X$, presents again that *"property"*. This question will be called the $T \oplus T$*-"property" problem.* In this talk we will discuss the $T \oplus T$-recurrence problem, which was posed as an open question in [4]. In particular, we will motivate the problem by looking at the respective $T \oplus T$-hypercyclicity version (see [5, 2, 3]); and then we will solve the $T \oplus T$-recurrence problem in the negative in every separable infinite-dimensional Banach space by modifying the construction given in [1].

This talk is based on a joint work with Sophie Grivaux and Alfred Peris [6].

Bibliography

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June 12
13:30-14:20

Dynamics of the Backward Taylor Shift on Bergman Spaces

Jürgen Müller

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For $p \geq 1$ and open sets Ω in the extended complex plane containing the origin, the backward Taylor shift on the Bergman space $A^p(\Omega)$ is defined by

$$(Tf)(z) := \begin{cases} (f(z) - f(0))/z, & z \neq 0 \\ f'(0), & z = 0 \end{cases}.$$

Depending on Ω and p, the operator shows a rich variety concerning its dynamical behaviour. Some aspects are discussed in the talk.

Amenability of Köthe sequence algebras of order zero and infinity

June 12
16:30-17:20

Krzysztof Piszczek
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The notion of *amenability* arose in group theory. Its origin can be driven back to the Banach-Tarski paradox. Nowadays we understand that the three-dimensional ball can be given a paradoxical decomposition because the special orthogonal group $SO(3)$ contains a subgroup isomorphic to the free group $\mathbb{F}_2$ in two generators and this last group is non-amenable. We will recall the definition (and its equivalent forms) of an amenable group and outline briefly examples of amenable and non-amenable ones. The main part of the talk will however be devoted to the theory of amenable algebras.

In 1972 Barry Johnson proved that a locally compact group G is amenable if and only if its convolution algebra $L^1(G)$ satisfies the property that all derivations into dual modules are inner. This last property serves now as a definition of an *amenable* Banach algebra. Together with amenability we will also focus on two other amenability-like properties, i.e. *contractibility* (a stronger condition) and *weak amenability* (a weaker condition). We will briefly discuss characterizations of amenability-like properties in few Banach algebra categories such as convolution algebras or C^*-algebras. Further on we will study these properties in greater detail within the class of Fréchet sequence algebras. Finally we will indicate new directions towards characterizations of amenable vector-valued sequence algebras.

On polynomial-like properties of differentiable functions

June 13
9:50-10:40

Armin Rainer
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In this talk, I will show that differentiable functions, defined on a convex body in Euclidean space, whose derivatives are controlled by a suitable given sequence of positive real numbers share many properties with polynomials. The role of the degree of a polynomial is played by an integer associated with the given sequence, the diameter of the domain, and the sup-norm of the function. The polynomial-like behaviour of controlled differentiable functions is manifested in quantitative information on the size of the zero set and its local parameterization by Sobolev functions, a Remez-type inequality, a comparison of L^p-norms (reversing Hölder's inequality), a log-BMO-like property, etc. The results depend only on the derivatives up to some finite order, which can be determined explicitly.
